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澳門四高校聯合入學考試 (語言科及數學科)

Joint Admission Examination for Macao Four Higher Education Institutions (Languages and Mathematics)

2024 試題及參考答案
2024 Examination Paper and Suggested Answer

數學正卷 Mathematics Standard Paper

<u>第一部份</u> 選擇題。請選出每題之最佳答案。

1. 設集合 $A=\{x:x^2-3x-4\leq 0\}$，$B=\{x:3x+a\geq 0\}$，且 $A\cap B=\{x:2\leq x\leq 4\}$，則 $a=($　　$)$。

   A. -12　　B. -6　　C. -3　　D. 6　　E. 12

2. 已知對於所有實數 x，$f(x)=f(x+1)+1$。如果 $f(0)=16$，那麼 $f(15)$ 的值是 (　　)。

   A. 0　　B. 1　　C. 15　　D. 16　　E. 17

3. 設 x 和 y 滿足 $4x+5y=x(y+1)-(x-1)(y-1)$。如果 x 的值增加 4，則 y 的值是 (　　)。

   A. 減少了 8　　B. 減少了 4　　C. 減少了 2
   D. 增加了 4　　E. 增加了 8

4. $(\sqrt{x}-2)^5(2x-1)^4$ 的展開式中 x 的係數為 (　　)。

   A. -182　　B. -178　　C. 176　　D. 178　　E. 184

5. $P(2,3)$ 是 x-y 坐標平面上的固定點。M 是一個移動點，與 P 點保持固定距離。如果 M 的軌跡經過原點，M 的軌跡方程是 (　　)。

   A. $x^2+y^2-13=0$　　B. $x^2+y^2+4x-6y=0$　　C. $x^2+y^2+4x+6y=0$
   D. $x^2+y^2-4x-6y=0$　　E. $x^2+y^2-4x-6y+13=0$

6. $\dfrac{3\log\frac{1}{2}+\log 16}{\log 4+\log 5-1}=($　　$)$。

   A. 1　　B. -1　　C. 2　　D. -2　　E. 4

7. 等比數列的第 2 項及第 5 項的和是 9，同時第 7 項及第 10 項的和為 288，則數列第 20 項的數值為 (　　)。

   A. 32768　　B. 65536　　C. 131072　　D. 262144　　E. 524288

8. 如果數據集 $\{n,n-3,2n+5,4n-4,5n+10\}$ 的算數平均值為 6.8，則它的中位數是 (　　)。

   A. 4　　B. 5　　C. 15　　D. 0　　E. -1

9. $\sqrt{1+\left(\dfrac{m^4-1}{2m^2}\right)^2} = (\qquad)$。

A. $\dfrac{m^4+2m+1}{2m^2}$ B. $\dfrac{m^4-1}{2m^2}$ C. $\dfrac{m^2}{2}+\dfrac{1}{2m^2}$ D. $\dfrac{\sqrt{m^2+1}}{2}$ E. 以上皆非

10. 在銳角三角形 $\triangle ABC$ 中，$|AB|=8$，$|AC|=7$，$\sin C=\dfrac{4\sqrt{3}}{7}$，則 $|BC|=(\qquad)$。

A. 6 B. 12 C. 2 D. 3 E. 5

11. 拋物線在 $(-2,0)$ 和 $(6,0)$ 與 x 軸相交，在 $(0,4)$ 與 y 軸相交。如果 (m,n) 是拋物線上的一點，n 的最大值是 (　　)。

A. $\dfrac{8}{3}$ B. $\dfrac{16}{3}$ C. 4 D. 8 E. 16

12. 在下列區間 (　　) 中，函數 $f(x)=5\cos(x+\dfrac{\pi}{3})$ 單調遞增。

A. $\left(0,\dfrac{\pi}{2}\right)$ B. $\left(\dfrac{\pi}{2},\pi\right)$ C. $\left(\pi,\dfrac{3\pi}{2}\right)$

D. $\left(\dfrac{3\pi}{2},2\pi\right)$ E. $\left(\dfrac{\pi}{3},\dfrac{5\pi}{6}\right)$

13. 若 $\theta\in[0,\pi)$ 且 $1+\sin\theta-2\cos^2\theta=0$，則 $\theta=(\qquad)$。

A. $\dfrac{\pi}{6}$ 或 $\dfrac{5\pi}{6}$ B. $\dfrac{\pi}{3}$ C. $\dfrac{\pi}{6}$ 或 $\dfrac{\pi}{3}$ D. $\dfrac{\pi}{6}$ 或 $\dfrac{\pi}{2}$ E. $\dfrac{\pi}{3}$ 或 $\dfrac{\pi}{2}$

14. 已知 $x^2-3x+1=0$，則 $x^4+\dfrac{1}{x^4}=(\qquad)$。

A. 2 B. 47 C. 49 D. 79 E. 81

15. 設函數 $f(x)$ 是定義域為 $\mathbb{R}$ 的偶函數，且在 $(-\infty,0)$ 單調遞減，則以下正確的是 (　　)。

A. $f(2^{-\frac{7}{3}})>f(3^{-\frac{2}{7}})>f(\log_3\frac{2}{7})$ B. $f(3^{-\frac{2}{7}})>f(\log_3\frac{2}{7})>f(2^{-\frac{7}{3}})$

C. $f(\log_3\frac{2}{7})>f(2^{-\frac{7}{3}})>f(3^{-\frac{2}{7}})$ D. $f(3^{-\frac{2}{7}})>f(2^{-\frac{7}{3}})>f(\log_3\frac{2}{7})$

E. $f(\log_3\frac{2}{7})>f(3^{-\frac{2}{7}})>f(2^{-\frac{7}{3}})$

<u>第二部份</u> 解答題。

1. 10 件產品中含有 3 件次品。現隨機抽出 4 件。

   (a) 求抽出至少有 2 件次品的概率。 (4 分)

   (b) 求抽出的次品數的數學期望。 (4 分)

2. 已知 $\alpha, \beta \in (0, \frac{\pi}{2})$，$\tan\alpha = \frac{1}{5}$，$\cos\beta = \frac{3\sqrt{13}}{13}$。

   (a) 求 $\tan(\alpha+\beta)$ 的值。 (4 分)

   (b) 求 $\cos(\alpha+2\beta)$ 的值。 (4 分)

3. 已知等差數列 $\{a_n\}_{n\geq 1}$ 中 $a_1 = 3$，並且 a_1，a_2 及 a_5 成等比數列。

   (a) 求 $\{a_n\}_{n\geq 1}$ 的通項公式。 (4 分)

   (b) 設 S_n 為 $\{a_n\}_{n\geq 1}$ 的前 n 項和，是否存在正整數 n 使得 $S_n \geq 12n + 36$？若存在，求 n 的最小值。若不存在，請說明理由。 (4 分)

4. 設函數 $f(x) = a - |x-3| - |x-7|$。

   (a) 當 $a = 8$ 時，求解不等式 $f(x) \geq 0$。 (4 分)

   (b) 如果 $g(x) = xf(x)$ 在閉區間 $[-1, 1]$ 上有最小值 -1，求 a 的值。 (4 分)

5. 已知雙曲線 $\mathcal{C} : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 的離心率為 $\frac{\sqrt{10}}{2}$，點 $A(2\sqrt{2}, 3)$ 在該雙曲線上。直線 $\ell : y = x + m$ 與 $\mathcal{C}$ 相交於 P 和 Q 兩點，且 $OP \perp OQ$，這裡 O 為坐標系原點。

   (a) 求雙曲線 $\mathcal{C}$ 的方程。 (3 分)

   (b) 求 m 的值。 (5 分)

參考答案

<u>第一部份</u> 選擇題。

題目編號	最佳答案
1	B
2	B
3	C
4	C
5	D
6	A
7	D
8	A
9	C
10	E
11	B
12	C
13	A
14	B
15	E

<u>第二部份</u> 解答題。

1. 設抽出的次品數為 X。

   (a) 抽出 4 件產品中恰有 2 件次品的概率為 $P(X=2)=\frac{{}_3C_2\cdot{}_7C_2}{{}_{10}C_4}=\frac{3}{10}$，抽出 4 件產品中恰有 3 件次品的概率為 $P(X=3)=\frac{{}_3C_3\cdot{}_7C_1}{{}_{10}C_4}=\frac{1}{30}$。因此抽出至少 2 件次品的概率為 $P(X\geq 2)=P(X=2)+P(X=3)=\frac{3}{10}+\frac{1}{30}=\frac{1}{3}$。

   (b) 抽出 4 件產品中全為正品的概率為 $P(X=0)=\frac{{}_3C_0\cdot{}_7C_4}{{}_{10}C_4}=\frac{1}{6}$，抽出 4 件產品中恰有 1 件次品的概率為 $P(X=1)=\frac{{}_3C_1\cdot{}_7C_3}{{}_{10}C_4}=\frac{1}{2}$。因此抽出的次品數的數學期望值 $E(X)=0\cdot P(X=0)+1\cdot P(X=1)+2\cdot P(X=2)+3\cdot P(X=3)=1\cdot\frac{1}{2}+2\cdot\frac{3}{10}+3\cdot\frac{1}{30}=\frac{6}{5}$。

2. 因為 $\beta\in(0,\dfrac{\pi}{2})$ 且 $\cos\beta=\dfrac{3\sqrt{13}}{13}$，所以 $\sin\beta=\sqrt{1-\cos^2\beta}=\dfrac{2\sqrt{13}}{13}$，$\tan\beta=\dfrac{2}{3}$。

   (a) 因為 $\tan\alpha=\dfrac{1}{5}$，所以 $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=1$。

   (b) 因為 $\alpha,\beta\in(0,\dfrac{\pi}{2})$，所以由 (a) 知，$\alpha+\beta=\dfrac{\pi}{4}$。於是 $\cos(\alpha+2\beta)=\cos[(\alpha+\beta)+\beta]=\cos(\beta+\dfrac{\pi}{4})=\dfrac{\sqrt{2}}{2}(\cos\beta-\sin\beta)=\dfrac{\sqrt{26}}{26}$。

3. (a) 設等差數列的公差為 d，因為 $a_1=3$，所以 $a_2=3+d,a_5=3+4d$。又因為 a_1，a_2 及 a_5 成等比數列，所以 $a_2^2=a_1a_5$，即 $(3+d)^2=3(3+4d)$，解得 $d=0$ 或 $d=6$。因此 $a_n=3$ 或 $a_n=3+6(n-1)=6n-3$。

   (b) 由等差數列前 n 項和公式知 $S_n=3n$ 或 $S_n=3n^2$。（1）當 $S_n=3n$ 時，不存在滿足題意的正整數 n；（2）當 $S_n=3n^2$ 時，只需要 $3n^2\geq 12n+36$，即 $n\geq 6$，因此滿足條件的 n 的最小值是 6。

4. (a) $f(x)\geq 0$ 意味著 $|x-3|+|x-7|\leq 8$。所以，x 到 3 和 7 的距離之和不超過 8。所以 $1\leq x\leq 9$。

   (b) $f(x)$ 定義在閉區間 $[-1,1]$ 上，所以 $f(x)=a-(3-x)-(7-x)=2x+(a-10)$ 以及 $g(x)=xf(x)=2x^2+(a-10)x$。$g(x)$ 的對稱軸為 $x_0=-\dfrac{a-10}{4}$。

   i.) 如果 $-\dfrac{a-10}{4}\in[-1,1]$，即 $a\in[6,14]$，那麼 $g(x)$ 的最小值為 $-\dfrac{(a-10)^2}{8}=-1$，得 $a=10\pm 2\sqrt{2}$。

ii.) 如果 $-\dfrac{a-10}{4} \geq 1$，即 $a \leq 6$，那麼 $g(x)$ 在 $x=1$ 處取得最小值，即 $2+(a-10)=a-8=-1$，解得 $a=7$。不符合。

iii.) 如果 $-\dfrac{a-10}{4} \leq -1$，即 $a \geq 14$，那麼 $g(x)$ 在 $x=-1$ 處取得最小值，即 $2-(a-10)=-1$，解得 $a=13$，不符合。

總之，$a=10\pm 2\sqrt{2}$。

5. (a) 點 A 在雙曲線上，所以 $\frac{8}{a^2}-\frac{9}{b^2}=1$。又因為離心率 $e=\frac{\sqrt{a^2+b^2}}{a}=\frac{\sqrt{10}}{2}$。容易求得 $a^2=2$，$b^2=3$。因此所求雙曲線方程為 $\frac{x^2}{2}-\frac{y^2}{3}=1$。

(b) 設直線 l 與雙曲線 $\mathcal{C}$ 的交點分別為 $P(x_1,y_1)$ 和 $Q(x_2,y_2)$。聯立直線 l 和雙曲線 $\mathcal{C}$ 的方程，我們有 $3x^2-2(x+m)^2-6=0$，即 $x^2-4mx-2m^2-6=0$。根據韋達定理，$x_1+x_2=4m$ 及 $x_1x_2=-2m^2-6$。又因為 $OP\perp OQ$，所以 $y_1y_2+x_1x_2=0$。注意到點 P 和點 Q 在直線 l 上，於是 $(x_1+m)(x_2+m)+x_1x_2=0$，整理得到 $2x_1x_2+m(x_1+x_2)+m^2=0$。由韋達定理的結論我們可得 $2(-2m^2-6)+4m^2+m^2=0$，解得 $m^2=12$，$m=\pm 2\sqrt{3}$。

<u>Part I</u> Multiple choice questions. Choose the *best answer* for each question.

1. Let sets $A = \{x : x^2 - 3x - 4 \leq 0\}$, $B = \{x : 3x + a \geq 0\}$ and $A \cap B = \{x : 2 \leq x \leq 4\}$, then $a = (\quad)$.

   A. -12 B. -6 C. -3 D. 6 E. 12

2. Given that $f(x) = f(x+1) + 1$ for all real numbers x and $f(0) = 16$, the value of $f(15)$ is $(\quad)$.

   A. 0 B. 1 C. 15 D. 16 E. 17

3. Suppose x and y satisfy $4x + 5y = x(y+1) - (x-1)(y-1)$. If the value of x is increased by 4, then the value of y is $(\quad)$.

   A. decreased by 8 B. decreased by 4 C. decreased by 2

   D. increased by 4 E. increased by 8

4. The coefficient of x in the expansion of $\left(\sqrt{x} - 2\right)^5 (2x-1)^4$ is $(\quad)$.

   A. -182 B. -178 C. 176 D. 178 E. 184

5. $P(2,3)$ is a fixed point on a x-y coordinate plane. M is a moving point such that it maintains a fixed distance from point P. If the locus of M passes through the origin, the equation of the locus of M is $(\quad)$.

   A. $x^2 + y^2 - 13 = 0$ B. $x^2 + y^2 + 4x - 6y = 0$ C. $x^2 + y^2 + 4x + 6y = 0$

   D. $x^2 + y^2 - 4x - 6y = 0$ E. $x^2 + y^2 - 4x - 6y + 13 = 0$

6. $\dfrac{3\log\frac{1}{2} + \log 16}{\log 4 + \log 5 - 1} = (\quad)$.

   A. 1 B. -1 C. 2 D. -2 E. 4

7. The sum of the 2nd term and the 5th term of a geometric sequence is 9, while the sum of the 7th term and 10th term of the sequence is 288, then the sequence of the 20th term is $(\quad)$.

   A. 32768 B. 65536 C. 131072 D. 262144 E. 524288

8. If the mean of the data set $\{n, n-3, 2n+5, 4n-4, 5n+10\}$ is 6.8, the median of this set is $(\quad)$.

   A. 4 B. 5 C. 15 D. 0 E. -1

9. $\sqrt{1+\left(\frac{m^4-1}{2m^2}\right)^2}=($ $\quad$ $)$.

A. $\frac{m^4+2m+1}{2m^2}$

B. $\frac{m^4-1}{2m^2}$

C. $\frac{m^2}{2}+\frac{1}{2m^2}$

D. $\frac{\sqrt{m^2+1}}{2}$

E. None of the above

10. In the acute triangle $\triangle ABC$, $|AB|=8$, $|AC|=7$, $\sin C=\frac{4\sqrt{3}}{7}$. Then $|BC|=($ $\quad$ $)$.

A. 6 B. 12 C. 2 D. 3 E. 5

11. A parabola cuts the x-axis at $(-2,0)$ and $(6,0)$ and the y-axis at $(0,4)$. If (m,n) is a point lying on the parabola, the maximum value of n is ($\quad$).

A. $\frac{8}{3}$ B. $\frac{16}{3}$ C. 4 D. 8 E. 16

12. In the following interval ($\quad$), the function $f(x)=5\cos(x+\frac{\pi}{3})$ increases monotonically.

A. $\left(0,\frac{\pi}{2}\right)$

B. $\left(\frac{\pi}{2},\pi\right)$

C. $\left(\pi,\frac{3\pi}{2}\right)$

D. $\left(\frac{3\pi}{2},2\pi\right)$

E. $\left(\frac{\pi}{3},\frac{5\pi}{6}\right)$

13. If $\theta\in[0,\pi)$ and $1+\sin\theta-2\cos^2\theta=0$, then $\theta=($ $\quad$ $)$.

A. $\frac{\pi}{6}$ or $\frac{5\pi}{6}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{6}$ or $\frac{\pi}{3}$ D. $\frac{\pi}{6}$ or $\frac{\pi}{2}$ E. $\frac{\pi}{3}$ or $\frac{\pi}{2}$

14. If $x^2-3x+1=0$, then $x^4+\frac{1}{x^4}=($ $\quad$ $)$.

A. 2 B. 47 C. 49 D. 79 E. 81

15. Let $f(x)$ be an even function defined on $\mathbb{R}$ and decrease in $(-\infty,0)$. Which of the following is true? ($\quad$).

A. $f(2^{-\frac{7}{3}})>f(3^{-\frac{2}{7}})>f(\log_3\frac{2}{7})$

B. $f(3^{-\frac{2}{7}})>f(\log_3\frac{2}{7})>f(2^{-\frac{7}{3}})$

C. $f(\log_3\frac{2}{7})>f(2^{-\frac{7}{3}})>f(3^{-\frac{2}{7}})$

D. $f(3^{-\frac{2}{7}})>f(2^{-\frac{7}{3}})>f(\log_3\frac{2}{7})$

E. $f(\log_3\frac{2}{7})>f(3^{-\frac{2}{7}})>f(2^{-\frac{7}{3}})$

Part II Problem-solving questions.

1. There are 3 defective products among the 10 products. Now 4 products are randomly selected.

   (a) Find the probability that at least 2 defective products are selected. (4 marks)

   (b) Find the expected number of defective products selected. (4 marks)

2. Given that $\alpha, \beta \in (0, \frac{\pi}{2})$, $\tan\alpha = \frac{1}{5}$, $\cos\beta = \frac{3\sqrt{13}}{13}$.

   (a) Find the value of $\tan(\alpha + \beta)$. (4 marks)

   (b) Find the value of $\cos(\alpha + 2\beta)$. (4 marks)

3. In the arithmetic series $\{a_n\}_{n\geq 1}$, $a_1 = 3$. a_1, a_2 and a_5 form a geometic series.

   (a) Find the general term for $\{a_n\}_{n\geq 1}$. (4 marks)

   (b) Let S_n be the nth partial sum of $\{a_n\}_{n\geq 1}$. Is there a positive integer n such that $S_n \geq 12n + 36$? If yes, find the smallest value of such n. If no, give your reason. (4 marks)

4. Suppose the function $f(x) = a - |x - 3| - |x - 7|$.

   (a) If $a = 8$, solve the inequality $f(x) \geq 0$. (4 marks)

   (b) If the function $g(x) = xf(x)$ has its minimum value -1 in the closed interval $[-1, 1]$, find the value of a. (4 marks)

5. Suppose that the eccentricity of the hyperbolic $\mathcal{C} : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{\sqrt{10}}{2}$. The point $A(2\sqrt{2}, 3)$ lies on the hyperbolic $\mathcal{C}$. The straight line $\ell : y = x + m$ intersects $\mathcal{C}$ at points P and Q and $OP \perp OQ$, where Q is the origin of the coordinate system.

   (a) Find the equation for $\mathcal{C}$. (3 marks)

   (b) Find the value of m. (5 marks)

Suggested Answer

<u>Part I</u> Multiple choice questions.

Question Number	Best Answer
1	B
2	B
3	C
4	C
5	D
6	A
7	D
8	A
9	C
10	E
11	B
12	C
13	A
14	B
15	E

<u>Part II</u> Problem-solving questions.

1. Let the number of defective items selected be X.

   (a) The probability of selecting exactly 2 defective items out of 4 is $P(X=2)=\frac{{}_3C_2\cdot{}_7C_2}{{}_{10}C_4}=\frac{3}{10}$, and the probability of selecting exactly 3 defective items out of 4 is $P(X=3)=\frac{{}_3C_3\cdot{}_7C_1}{{}_{10}C_4}=\frac{1}{30}$. Therefore, the probability of selecting at least 2 defective items is $P(X\geq 2)=P(X=2)+P(X=3)=\frac{3}{10}+\frac{1}{30}=\frac{1}{3}$.

   (b) The probability of selecting 4 non-defective items is $P(X=0)=\frac{{}_3C_0\cdot{}_7C_4}{{}_{10}C_4}=\frac{1}{6}$, and the probability of selecting exactly 1 defective item out of 4 is $P(X=1)=\frac{{}_3C_1\cdot{}_7C_3}{{}_{10}C_4}=\frac{1}{2}$. Therefore, the expected number of defective products selected is $E(X)=0\cdot P(X=0)+1\cdot P(X=1)+2\cdot P(X=2)+3\cdot P(X=3)=1\cdot\frac{1}{2}+2\cdot\frac{3}{10}+3\cdot\frac{1}{30}=\frac{6}{5}$.

2. Since $\beta\in(0,\dfrac{\pi}{2})$ and $\cos\beta=\dfrac{3\sqrt{13}}{13}$, therefore $\sin\beta=\sqrt{1-\cos^2\beta}=\dfrac{2\sqrt{13}}{13}$ and $\tan\beta=\dfrac{2}{3}$.

   (a) Since $\tan\alpha=\dfrac{1}{5}$, we have $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=1$.

   (b) Since $\alpha,\beta\in(0,\dfrac{\pi}{2})$, by (a), $\alpha+\beta=\dfrac{\pi}{4}$. Then $\cos(\alpha+2\beta)=\cos[(\alpha+\beta)+\beta]=\cos(\beta+\dfrac{\pi}{4})=\dfrac{\sqrt{2}}{2}(\cos\beta-\sin\beta)=\dfrac{\sqrt{26}}{26}$.

3. (a) Let the common difference of the arithmetic sequence be d. Since $a_1=3$, we have $a_2=3+d$ and $a_5=3+4d$. Because a_1, a_2, and a_5 form a geometric sequence, we have $a_2^2=a_1a_5$, that is, $(3+d)^2=3(3+4d)$. Solving this equation, we find $d=0$ or $d=6$. Therefore, $a_n=3$ or $a_n=3+6(n-1)=6n-3$.

   (b) From the formula for the sum of the first n terms of an arithmetic sequence, we know $S_n=3n$ or $S_n=3n^2$. (1) When $S_n=3n$, there is no positive integer n that satisfies the given conditions. (2) When $S_n=3n^2$, we require $3n^2\geq 12n+36$, which implies $n\geq 6$. Therefore, the smallest positive integer n that satisfies the condition is 6.

4. (a) $f(x)\geq 0$ implies $|x-3|+|x-7|\leq 8$. Then, the sum of the distances from x to 3 and 7 does not exceed 8. Thus $1\leq x\leq 9$.

(b) $f(x)$ is defined on the closed interval $[-1, 1]$. Thus $f(x) = a - (3 - x) - (7 - x) = 2x + (a - 10)$ and $g(x) = xf(x) = 2x^2 + (a - 10)x$. The axis of symmetry for $g(x)$ is $x_0 = -\dfrac{a - 10}{4}$.

i.) If $-\dfrac{a - 10}{4} \in [-1, 1]$, i.e., $a \in [6, 14]$, then the minimum value of $g(x)$ is $-\dfrac{(a - 10)^2}{8} = -1$, giving $a = 10 \pm 2\sqrt{2}$.

ii.) If $-\dfrac{a - 10}{4} \geq 1$, i.e., $a \leq 6$, then $g(x)$ achieves its minimum value at $x = 1$. That is, $2+(a-10) = a - 8 = -1$, and $a = 7$. This does not satisfy the condition.

iii.) If $-\dfrac{a - 10}{4} \leq -1$, i.e., $a \geq 14$, then $g(x)$ achieves its minimum value at $x = -1$. That is $2 - (a - 10) = -1$, and $a = 13$. This does not satisfy the condition.

In conclusion, $a = 10 \pm 2\sqrt{2}$.

5. (a) Point A lies on the hyperbola, so $\frac{8}{a^2} - \frac{9}{b^2} = 1$. Also, since the eccentricity $e = \frac{\sqrt{a^2+b^2}}{a} = \frac{\sqrt{10}}{2}$, it is easy to find $a^2 = 2$ and $b^2 = 3$. Therefore, the equation of the hyperbola is $\frac{x^2}{2} - \frac{y^2}{3} = 1$.

(b) Suppose the intersection points of the line l and the hyperbola $\mathcal{C}$ are $P(x_1, y_1)$ and $Q(x_2, y_2)$. By solving the equations of line l and hyperbola $\mathcal{C}$ together, we have $3x^2 - 2(x + m)^2 - 6 = 0$, i.e., $x^2 - 4mx - 2m^2 - 6 = 0$. According to Vieta's formulas, $x_1 + x_2 = 4m$ and $x_1x_2 = -2m^2 - 6$. Since $OP \perp OQ$, we have $y_1y_2 + x_1x_2 = 0$. Noting that points P and Q lie on the line l, we get $(x_1 + m)(x_2 + m) + x_1x_2 = 0$. Simplifying, we obtain $2x_1x_2 + m(x_1 + x_2) + m^2 = 0$. From Vieta's formulas, we have $2(-2m^2 - 6) + 4m^2 + m^2 = 0$, which gives $m^2 = 12$, so $m = \pm 2\sqrt{3}$.