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澳門科技大學
UNIVERSIDADE DE CIÊNCIA E TECNOLOGIA DE MACAU
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澳門四高校聯合入學考試 (語言科及數學科)

Joint Admission Examination for Macao Four Higher Education Institutions (Languages and Mathematics)

2021 試題及參考答案
2021 Examination Paper and Suggested Answer

數學正卷 Mathematics Standard Paper

第一部份 選擇題。請選出每題之*最佳答案*。

1. 若集合 $P=\{1, 2, 3, 5, 7, 11\}$，$Q=\{x: x^2-15x+36<0\}$，則 $P\cap Q$ 中元素的個數為

A. 2　B. 3　C. 4　D. 5　E. 1

2. 已知瑪麗和約翰兩人分別要用 4 小時及 3 小時完成某件工作。若他們一起做，需要多少小時才能完成 5 件相同的工作？

A. 32/5　B. 35/4　C. 35/2　D. 20/3　E. 60/7

3. 有一等差數列前 n 項之總和為 n^2，求此數列的第 10 項。

A. 19　B. 21　C. 28　D. 31　E. 40

4. 若對所有實數 x，$y=mx^2+6x+3m$ 都為正數，求 m 的取值範圍。

A. $0<m<\sqrt{3}$　B. $m>\sqrt{3}$　C. $-\sqrt{3}<m<\sqrt{3}$

D. $-\sqrt{3}<m<0$　E. 以上皆非

5. 已知 $-2x^2+3x-7=0$ 的根為 α 和 β。下列哪一個方程的根為 $\frac{1}{\alpha}$ 和 $\frac{1}{\beta}$？

A. $x^2-3x+7=0$　B. $7x^2-3x+2=0$　C. $7x^2+3x+2=0$

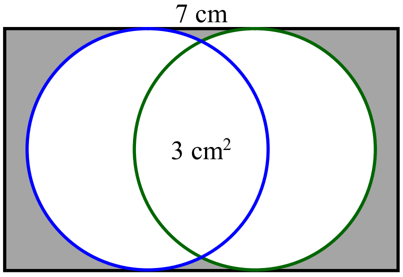
D. $2x^2-3x-7=0$　E. 以上皆非

6. 右圖中兩個半徑相等的圓與長方形的上、下邊相切。若長方形的長和寬分別為 7 cm 及 4 cm，而兩個圓的相交部份有面積 3 cm^2，陰影部份面積為多少 cm^2？

A. $31-8\pi$　B. $27-8\pi$

C. $27-4\pi$　D. $21-4\pi$

E. 以上皆非



7. 若方程式 $9^{-x^2}-4\cdot 3^{-x^2}=k$ 有實數解，下列哪個一定成立？

A. $k>0$　B. $-4\le k\le 1$　C. $-3\le k<0$　D. $0<k\le 3$　E. 以上皆非

8. 設 $f(x)=-16x^3-mx-m$。若 $f(x)$ 能被 $2x+1$ 整除，求 m 之值。

A. -1　B. 1　C. 2　D. 4　E. 6

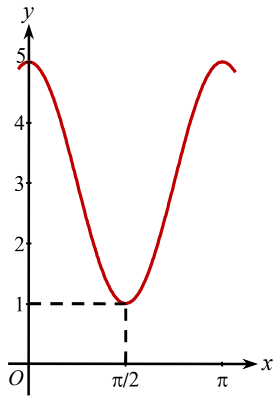
9. 103^{10} 的十位數字 (右面起計第二個數字；例如 43128 的十位數字是 2) 是

A. 2　B. 3　C. 4　D. 7　E. 以上皆非

10. 若 $\log_4 x = y - 3$ 及 $2(\log_4 x)^2 = 4 - y$，則 $x =$

A. $\frac{1}{4}$ 或 2　　B. $\frac{1}{2}$ 或 4　　C. $\frac{7}{2}$ 或 2　　D. $\frac{1}{4}$ 或 $\frac{7}{2}$　　E. 2 或 4

11. 右圖中所示為 ＿＿＿＿＿＿ 的圖像。



A. $y = 3 + 2\cos\frac{x}{2}$

B. $y = 3 + 2\cos 2x$

C. $y = 3 + 2\cos x$

D. $y = 1 + 2\cos\frac{x}{2}$

E. $y = 1 + 2\cos 2x$

12. 已知點 $P(-1, -3)$ 和 $Q(5, -1)$，則 PQ 的垂直平分線的方程為

A. $x + 3y - 4 = 0$　　B. $x - 3y + 4 = 0$　　C. $x + 3y + 4 = 0$
D. $3x - y - 4 = 0$　　E. $3x + y - 4 = 0$

13. 若一組數據 $x_1, x_2, \cdots, x_n$ 的平均數和方差分別為 1 和 0.01，則數據 $10x_1, 10x_2, \cdots, 10x_n$ 的平均數和方差分別是

A. 1 和 0.01　　B. 10 和 0.1　　C. 1 和 1　　D. 10 和 1　　E. 100 和 1

14. $\dfrac{\sqrt{140} - \sqrt{132}}{\sqrt{35} + \sqrt{33}} =$

A. $68 - 2\sqrt{1155}$　　B. $68 - \sqrt{1155}$　　C. $(34 - \sqrt{1155})/2$
D. $34 - \sqrt{1155}$　　E. $68 + 2\sqrt{1155}$

15. 若 $\frac{5}{a} + \frac{4}{b} = 3$ $(a, b > 0)$，則 ab 的最小值為

A. $\frac{20}{9}$　　B. $\frac{20}{3}$　　C. $\frac{80}{9}$　　D. $\frac{80}{3}$　　E. $\frac{\sqrt{20}}{3}$

第二部份 **解答題。**

1. 書架上有中文書 4 本、英文書 2 本、數學書 3 本。

   (a) 從這書架上隨機地選取 3 本書。求取得中、英、數各一本的概率。 (3 分)

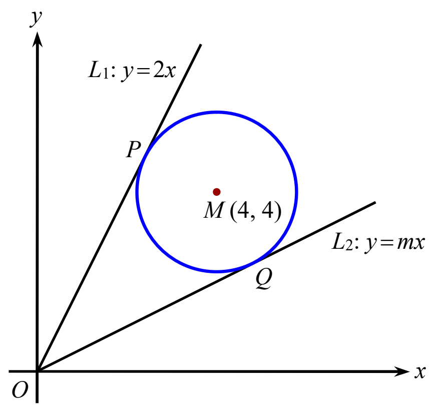
   (b) 將這九本書隨機地重新排列。求同類書籍排在一起的概率。 (5 分)

   [注：以最簡分數表示 (a) 和 (b) 的答案。]

2. 右圖中，一個以 $M\ (4,\ 4)$ 為圓心的圓與 $L_1: y=2x$ 和 $L_2: y=mx$ 兩條直綫相切。兩條直綫對圓的切點分別為 P 和 Q。

   (a) 求圓的方程。 (4 分)

   (b) 求 m 的值。 (4 分)



3. 若 $x, y>0$ 及 $y^2-2myx+x^2=a^2$，其中 a 和 m 為常數，且 $0<m<1$。

   (a) 証明 $(1-m^2)y^2=a^2-(x-my)^2$。 (3 分)

   (b) 証明當 $y=\dfrac{x}{m}$ 時 y 值達至最大。 (3 分)

   (c) 由此決定 x 值 (以 a 和 m 表示) 使 y 值達至最大。 (2 分)

4. 在等差數列 $\{a_n\}_{n\geq 1}$ 中，已知 $a_2=3$ 及 $a_{20}=39$。

   (a) 求數列 $\{a_n\}_{n\geq 1}$ 的通項。 (3 分)

   (b) 設數列 $\left\{\dfrac{1}{a_n a_{n+1}}\right\}_{n\geq 1}$ 的前 n 項和為 S_n。若 $S_n=\dfrac{10}{21}$，求 n 的值。 (5 分)

5. 在 $\triangle ABC$ 中，$\sin(C-A)=1$ 及 $\cos B=\dfrac{2\sqrt{2}}{3}$。

   (a) 求 $\sin^2 C$。 (4 分)

   (b) 若 $|AC|=5$，求 $\triangle ABC$ 的面積。 (4 分)

JM01 數學正卷 (A 卷) – 參考答案

第一部份 選擇題。

題目編號	最佳答案
1	B
2	E
3	A
4	B
5	B
6	A
7	C
8	D
9	C
10	A
11	B
12	E
13	D
14	A
15	C

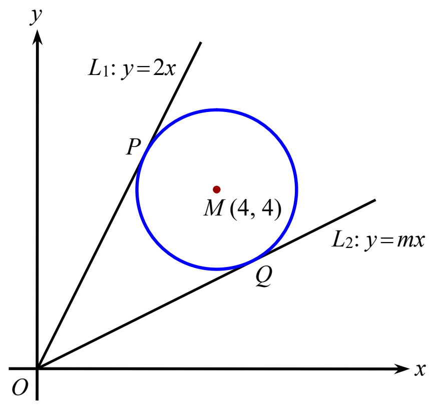
(第二部份答案由下頁開始)

第二部份 解答題。

1. (a) 這裏要從 9 本書取出 3 本，因此樣本空間的大小為 ${}_9C_3$。事件的大小為 $4\cdot2\cdot3$，故此所求概率為 $\frac{4\cdot2\cdot3}{{}_9C_3}=\frac{24}{84}=\frac{2}{7}$ 。

(b) 這裏要排列 9 本書，因此樣本空間的大小為 9!。此處的事件可視作為將三類書來排列，而中英數三類書各自有 4!、2!、3! 種方式來排列，故此所求概率為 $\frac{3!4!2!3!}{9!}=\frac{1}{210}$ 。

2. (a) 設 r 為圓的半徑。



方法一

圓的方程是 $(x-4)^2+(y-4)^2=r^2$ ---------------------- (1)

把 $y=2x$ 代入 (1)，我們有 $(x-4)^2+(2x-4)^2=r^2$ ，即

$$5x^2-24x+(32-r^2)=0 \text{ ---------------------- (2)}$$

在 L_1 與圓的切點 (即 P)，(2) 的判別式等於零，即

$$(-24)^2-4(5)(32-r^2)=0 \Leftrightarrow r^2=\frac{16}{5} 。$$

把最後一式代入 (1)，化簡後得圓的方程為

$$5x^2+5y^2-40x-40y+144=0 \text{ ------------- (3)}$$

方法二

L_1 的方程可寫為 $2x-y=0$。 $\therefore r=|MP|=\frac{|2(4)-4|}{\sqrt{4+1}}=\frac{4}{\sqrt{5}}$ ，從而圓的方程為

$$(x-4)^2+(y-4)^2=\frac{16}{5} \Leftrightarrow 5x^2+5y^2-40x-40y+144=0 。$$

(b) 方法一

把 $y=mx$ 代入 (3)，我們有 $5x^2+5(mx)^2-40x-40(mx)+144=0$ ，即

$$5(1+m^2)x^2-40(1+m)x+144=0 \text{ ----------- (4)}$$

在 L_2 與圓的切點 (即 Q)，(4) 的判別式等於零，即 $[-40(1+m)]^2-4(5)(1+m^2)(144)=0$ ，亦即 $2m^2-5m+2=0 \Leftrightarrow (m-2)(2m-1)=0$ 。對應於直綫 L_2， $m=\frac{1}{2}$ 。

方法二

直綫 $y=x$ 穿過 O 和 M。利用 OP 和 OQ 關於 $y=x$ 的對稱性，即得 $m=\frac{1}{L_1\text{的斜率}}=\frac{1}{2}$ 。

3. (a) 右邊 $= a^2-(x^2-2mxy+m^2y^2)=(a^2-x^2+2mxy)-m^2y^2=y^2-m^2y^2=$ 左邊。

(b) 由 (a) 的等式得

$$y^2=\frac{a^2}{1-m^2}-\frac{(x-my)^2}{1-m^2} \text{ ------------------------------ } (1)$$

由於 $0<m<1$，因此 $0<m^2<1$，從而 $\frac{a^2}{1-m^2}\geq 0$ 及 $\frac{(x-my)^2}{1-m^2}\geq 0$ 對任何實數 x,y 都成立。

故此從 (1) 知道對任何實數 x,y 都有 $y^2\leq\frac{a^2}{1-m^2}$，即 $\frac{a^2}{1-m^2}$ 為 y^2 的最大值，並且在 $x-my=0$ ($\Leftrightarrow y=\frac{x}{m}$) 時 y^2 達至這個最大值。

$\because y>0$，$\therefore$ 當 $y=\frac{x}{m}$ 時 y 值也達至最大。

(c) 設 $y_{\max}$ 代表 y 的最大值，並設當 $x=x_0$ 時 y 達至最大值。

由 (b) 得知 $y_{\max}=\sqrt{\frac{a^2}{1-m^2}}=\frac{|a|}{\sqrt{1-m^2}}$，並且 $x_0=my_{\max}=\frac{m|a|}{\sqrt{1-m^2}}$。

4. (a) 公差 $d=(a_{20}-a_2)/(20-2)=2$。

$\therefore a_1=a_2-d=1$。

$\therefore a_n=a_1+(n-1)d=2n-1$。

(b) $\because S_n=\sum_{k=1}^{n}\frac{1}{(2k-1)(2k+1)}=\sum_{k=1}^{n}\frac{1}{2}\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right)=\frac{1}{2}\left(\sum_{k=1}^{n}\frac{1}{2k-1}-\sum_{k=1}^{n}\frac{1}{2k+1}\right)=\frac{1}{2}\left(1-\frac{1}{2n+1}\right)=\frac{n}{2n+1}$，

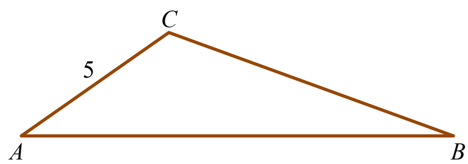
$\therefore S_n=\frac{10}{21}\Rightarrow\frac{n}{2n+1}=\frac{10}{21}\Rightarrow 21n=20n+10\Rightarrow n=10$。

5. (a) 由 $\sin(C-A)=1$ 及 $\cos B=\frac{2\sqrt{2}}{3}$，得 $A=C-90°$ 及 $\sin B=\frac{1}{3}$。

$\because A+B+C=180°$ 及 $A=C-90°$，$\therefore 2C=270°-B\Rightarrow\cos 2C=\cos(270°-B)=-\sin B=-\frac{1}{3}$。

$\therefore \sin^2 C=\frac{1-\cos 2C}{2}=\frac{1}{2}\left(1+\frac{1}{3}\right)=\frac{2}{3}$。

(b) 由 $C>90°$ 及 (a) 得 $\cos C=-\sqrt{1-\sin^2 C}=-\frac{\sqrt{3}}{3}$，從而有



$\sin A=\sin(C-90°)=-\cos C=\frac{\sqrt{3}}{3}$。

由正弦定理得 $BC=\frac{\sin A}{\sin B}AC=\frac{\sqrt{3}}{3}\cdot 3\cdot 5=5\sqrt{3}$。

$\therefore \triangle ABC$ 的面積 $=\frac{1}{2}AC\cdot BC\cdot\sin C=\frac{1}{2}\cdot 5\cdot 5\sqrt{3}\cdot\frac{\sqrt{2}}{\sqrt{3}}=\frac{25\sqrt{2}}{2}$。

Part I **Multiple choice questions. Choose the *best answer* for each question.**

1. If sets $P = \{1, 2, 3, 5, 7, 11\}$ and $Q = \{x: x^2 - 15x + 36 < 0\}$, then the number of elements in $P \cap Q$ is

   A. 2 B. 3 C. 4 D. 5 E. 1

2. Mary and John need respectively 4 hours and 3 hours to finish a certain task. If they work together, how many hours do they need to finish 5 such tasks?

   A. 32/5 B. 35/4 C. 35/2 D. 20/3 E. 60/7

3. The sum of the first n terms of an arithmetic sequence is n^2. Find the 10^{th} term of the sequence.

   A. 19 B. 21 C. 28 D. 31 E. 40

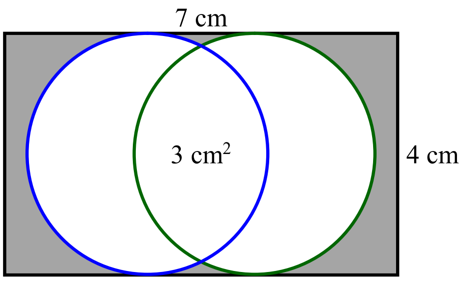
4. If $y = mx^2 + 6x + 3m$ is positive for any real number x, find the range of m.

   A. $0 < m < \sqrt{3}$ B. $m > \sqrt{3}$ C. $-\sqrt{3} < m < \sqrt{3}$
   D. $-\sqrt{3} < m < 0$ E. none of the above

5. Suppose α and β are the roots of $-2x^2 + 3x - 7 = 0$. Which of the following equations has $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ as its roots?

   A. $x^2 - 3x + 7 = 0$ B. $7x^2 - 3x + 2 = 0$ C. $7x^2 + 3x + 2 = 0$
   D. $2x^2 - 3x - 7 = 0$ E. none of the above

6. In the right figure, two identical circles touch the upper and lower edges of the rectangle. The length and width of the rectangle are 7 cm and 4 cm respectively, and the common portion of the two circles has area 3 cm^2. What is the area (in cm^2) of the shaded region?



   A. $31 - 8\pi$ B. $27 - 8\pi$
   C. $27 - 4\pi$ D. $21 - 4\pi$
   E. none of the above

7. Suppose the equation $9^{-x^2} - 4 \cdot 3^{-x^2} = k$ has a real root. Which of the following must be true?

   A. $k > 0$ B. $-4 \le k \le 1$ C. $-3 \le k < 0$ D. $0 < k \le 3$ E. none of the above

8. Let $f(x) = -16x^3 - mx - m$. If $f(x)$ is divisible by $2x + 1$, find the value of m.

   A. -1 B. 1 C. 2 D. 4 E. 6

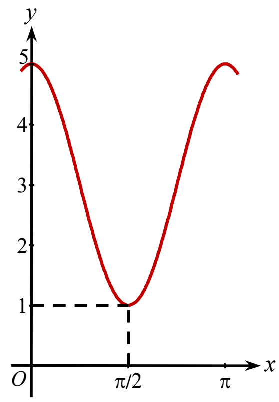
9. The tens digit (second last digit from the right; e.g. the tens digit of 43128 is 2) of 103^{10} is

   A. 2 B. 3 C. 4 D. 7 E. none of the above

10. If $\log_4 x = y - 3$ and $2(\log_4 x)^2 = 4 - y$, then $x =$

A. $\frac{1}{4}$ or 2 B. $\frac{1}{2}$ or 4 C. $\frac{7}{2}$ or 2 D. $\frac{1}{4}$ or $\frac{7}{2}$ E. 2 or 4

11. The right figure shows the graph of __________.



A. $y = 3 + 2\cos\frac{x}{2}$

B. $y = 3 + 2\cos 2x$

C. $y = 3 + 2\cos x$

D. $y = 1 + 2\cos\frac{x}{2}$

E. $y = 1 + 2\cos 2x$

12. $P(-1, -3)$ and $Q(5, -1)$ are two given points. The equation of the perpendicular bisector of PQ is

A. $x + 3y - 4 = 0$ B. $x - 3y + 4 = 0$ C. $x + 3y + 4 = 0$

D. $3x - y - 4 = 0$ E. $3x + y - 4 = 0$

13. For the data set $x_1, x_2, \cdots, x_n$, the mean and the variance are respectively 1 and 0.01. The mean and the variance for the data set $10x_1, 10x_2, \cdots, 10x_n$ are respectively

A. 1 and 0.01 B. 10 and 0.1 C. 1 and 1 D. 10 and 1 E. 100 and 1

14. $\dfrac{\sqrt{140} - \sqrt{132}}{\sqrt{35} + \sqrt{33}} =$

A. $68 - 2\sqrt{1155}$ B. $68 - \sqrt{1155}$ C. $(34 - \sqrt{1155})/2$

D. $34 - \sqrt{1155}$ E. $68 + 2\sqrt{1155}$

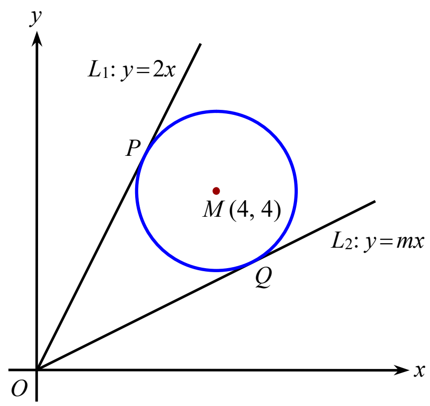
15. If $\frac{5}{a} + \frac{4}{b} = 3$ $(a, b > 0)$, then the minimum value of ab is

A. $\dfrac{20}{9}$ B. $\dfrac{20}{3}$ C. $\dfrac{80}{9}$ D. $\dfrac{80}{3}$ E. $\dfrac{\sqrt{20}}{3}$

Part II Problem-solving questions.

1. There are 4 Chinese books, 2 English books, and 3 Mathematics books on a bookshelf.
   (a) Three books are randomly chosen from the bookshelf. Find the probability that one Chinese book, one English book, and one Mathematics book are chosen. (3 marks)
   (b) These nine books are randomly re-arranged. Find the probability that books of the same kind are put together. (5 marks)
   [Note: Write the answers of (a) and (b) as fractions in the lowest terms.]

2. In the right figure, the two lines $L_1 : y = 2x$ and $L_2 : y = mx$ are tangent to a circle centered at $M\,(4, 4)$ at P and Q respectively.
   (a) Find the equation of the circle. (4 marks)
   (b) Find the value of m. (4 marks)



3. Suppose $x, y > 0$ and $y^2 - 2myx + x^2 = a^2$, where a and m are constants with $0 < m < 1$.
   (a) Show that $(1-m^2)y^2 = a^2 - (x-my)^2$. (3 marks)
   (b) Show that y attains its maximum value when $y = \frac{x}{m}$. (3 marks)
   (c) Hence determine the value of x (in terms of a and m) that maximizes y. (2 marks)

4. For the arithmetic sequence $\{a_n\}_{n\geq 1}$, $a_2 = 3$ and $a_{20} = 39$.
   (a) Find the general term of $\{a_n\}_{n\geq 1}$. (3 marks)
   (b) Let S_n be the sum of the first n terms of sequence $\left\{\frac{1}{a_n a_{n+1}}\right\}_{n\geq 1}$. If $S_n = \frac{10}{21}$, find the value of n. (5 marks)

5. In $\triangle ABC$, $\sin(C-A) = 1$ and $\cos B = \frac{2\sqrt{2}}{3}$.
   (a) Find $\sin^2 C$. (4 marks)
   (b) If $|AC| = 5$, find the area of $\triangle ABC$. (4 marks)

JM0l Mathematics Standard Paper A – Suggested Answer

<u>Part I</u> Multiple choice questions.

Question Number	Best Answer
1	B
2	E
3	A
4	B
5	B
6	A
7	C
8	D
9	C
10	A
11	B
12	E
13	D
14	A
15	C

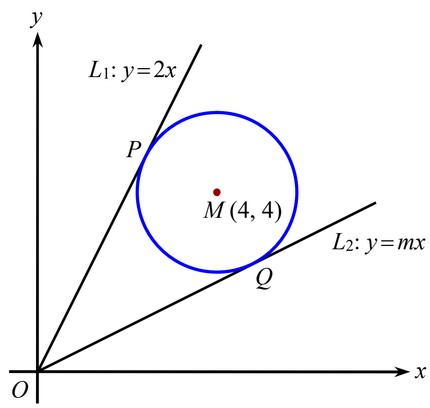
(Answers for Part II start from next page)

<u>Part II</u> Problem-solving questions.

1. (a) Here three books are chosen from nine, and so the size of the sample space is ${}_9C_3$. The size of the event is $4\cdot 2\cdot 3$, and hence the required probability is $\frac{4\cdot 2\cdot 3}{{}_9C_3}=\frac{24}{84}=\frac{2}{7}$.

   (b) Here we want to permute 9 books, and so the size of the sample space is 9!. The event can be regarded as a permutation of three groups, and there are 4! ways, 2! ways, and 3! ways to permute the Chinese books, English books, and Mathematics books respectively. It follows that the required probability is $\frac{3!4!2!3!}{9!}=\frac{1}{210}$.

2. (a) Let r be the radius of the circle.



<u>Method 1</u>

Equation of the circle is $(x-4)^2+(y-4)^2=r^2$ -------- (1)

Putting $y=2x$ into (1), we have $(x-4)^2+(2x-4)^2=r^2$, i.e.

$$5x^2-24x+(32-r^2)=0 \quad \text{---------------------- (2)}$$

At the tangent point P, the discriminant of (2) is zero, i.e.

$$(-24)^2-4(5)(32-r^2)=0 \Leftrightarrow r^2=\frac{16}{5}.$$

Putting the last equation into (1), upon simplification, we get the equation of the circle:

$$5x^2+5y^2-40x-40y+144=0 \quad \text{------------- (3)}$$

<u>Method 2</u>

Equation of L_1 can be written as $2x-y=0$. $\therefore$ $r=|MP|=\frac{|2(4)-4|}{\sqrt{4+1}}=\frac{4}{\sqrt{5}}$, and so the equation of the circle is

$$(x-4)^2+(y-4)^2=\frac{16}{5} \Leftrightarrow 5x^2+5y^2-40x-40y+144=0.$$

   (b) <u>Method 1</u>

Putting $y=mx$ into (3), we have $5x^2+5(mx)^2-40x-40(mx)+144=0$, i.e.

$$5(1+m^2)x^2-40(1+m)x+144=0 \quad \text{----------- (4)}$$

At the tangent point Q, the discriminant of (2) is zero, i.e. $[-40(1+m)]^2-4(5)(1+m^2)(144)=0 \Leftrightarrow$ $2m^2-5m+2=0 \Leftrightarrow (m-2)(2m-1)=0$. Corresponding to L_2, $m=\frac{1}{2}$.

<u>Method 2</u>

The line $y=x$ passes through O and M. Symmetry of OP and OQ about $y=x$ yields $m=\frac{1}{\text{Slope of } L_1}=\frac{1}{2}$.

3. (a) RHS $= a^2-(x^2-2mxy+m^2y^2)=(a^2-x^2+2mxy)-m^2y^2=y^2-m^2y^2=$ LHS.

(b) From the equality of (a) we have

$$y^2=\frac{a^2}{1-m^2}-\frac{(x-my)^2}{1-m^2} \quad \text{-------------------------------} \quad (1)$$

Since $0<m<1$, we have $0<m^2<1$, and so $\frac{a^2}{1-m^2}\geq 0$ and $\frac{(x-my)^2}{1-m^2}\geq 0$ is true for all $x, y\in\mathbb{R}$.

It follows from (1) that for all $x, y\in\mathbb{R}$, $y^2\leq\frac{a^2}{1-m^2}$, i.e. $\frac{a^2}{1-m^2}$ is the maximum value of y^2, and y^2 attains this maximum value when $x-my=0$ ($\Leftrightarrow$ $y=\frac{x}{m}$).

$\because$ $y>0$, $\therefore$ y also attains this maximum value when $y=\frac{x}{m}$.

(c) Let $y_{\max}$ represent the maximum value of y, and let y attain this maximum value when $x=x_0$.

From (b), $y_{\max}=\sqrt{\frac{a^2}{1-m^2}}=\frac{|a|}{\sqrt{1-m^2}}$ and $x_0=my_{\max}=\frac{m|a|}{\sqrt{1-m^2}}$.

4. (a) Common difference $d=(a_{20}-a_2)/(20-2)=2$.

$\therefore$ $a_1=a_2-d=1$.

$\therefore$ $a_n=a_1+(n-1)d=2n-1$.

(b) $\because$ $S_n=\sum_{k=1}^{n}\frac{1}{(2k-1)(2k+1)}=\sum_{k=1}^{n}\frac{1}{2}\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right)=\frac{1}{2}\left(\sum_{k=1}^{n}\frac{1}{2k-1}-\sum_{k=1}^{n}\frac{1}{2k+1}\right)=\frac{1}{2}\left(1-\frac{1}{2n+1}\right)=\frac{n}{2n+1}$,

$\therefore$ $S_n=\frac{10}{21}\Rightarrow\frac{n}{2n+1}=\frac{10}{21}\Rightarrow 21n=20n+10\Rightarrow n=10$.

5. (a) From $\sin(C-A)=1$ and $\cos B=\frac{2\sqrt{2}}{3}$, we have $A=C-90°$ and $\sin B=\frac{1}{3}$.

$\because$ $A+B+C=180°$ and $A=C-90°$, $\therefore$ $2C=270°-B\Rightarrow\cos 2C=\cos(270°-B)=-\sin B=-\frac{1}{3}$.

$\therefore$ $\sin^2 C=\frac{1-\cos 2C}{2}=\frac{1}{2}\left(1+\frac{1}{3}\right)=\frac{2}{3}$.

(b) From $C>90°$ and (a), we get $\cos C=-\sqrt{1-\sin^2 C}=-\frac{\sqrt{3}}{3}$,

and so $\sin A=\sin(C-90°)=-\cos C=\frac{\sqrt{3}}{3}$.

Law of Sines yields $BC=\frac{\sin A}{\sin B}AC=\frac{\sqrt{3}}{3}\cdot 3\cdot 5=5\sqrt{3}$.

$\therefore$ area of $\triangle ABC=\frac{1}{2}AC\cdot BC\cdot\sin C=\frac{1}{2}\cdot 5\cdot 5\sqrt{3}\cdot\frac{\sqrt{2}}{\sqrt{3}}=\frac{25\sqrt{2}}{2}$.