



INSTITUTE FOR TOURISM STUDIES

旅遊學院

Admission Examination 2014/2015

2014/2015 年度入學考試

Mathematics 數科

22/03/2014

Time allowed: Two hours

考試時間：二小時

Total Marks: 100

總分：100

1. This examination consists of two parts: (Part I) 5 work problems and (Part II) 15 multiple-choice questions (a total of 14 pages). Answer all of them.
此份試卷共有兩部份：(第一部份) 5 題答題 和 (第二部份) 15 題選擇題 (共14頁)，請全部作答。
2. (Part I) In order to obtain full credit in each question, you have to show all the steps in your calculations leading to a correct answer in the space provided in this booklet.
(第一部份) 如想取得滿分，必須於此試卷內每個問題下所提供之空白位置詳細列明計算步驟。
3. (Part II) You are required to record your answer by clearly circle **one and only one** of the five alternatives A, B, C, D or E that corresponds to your solution
(第二部份) 每一題所列出的五個答案中，只有一個答案是正確的，請選擇**圈出一個**正確的答案。
4. (Part II) The grading scheme will be as follows:
(第二部份) 評分標準如下：

Correct answer: +5 points , Incorrect answer: 0 point , No response: 0 point.
正確答案：加 5 分， 錯誤答案：零分， 不作答：零分。
5. You may use a non-programmable calculator
可使用沒有設定方程式的計算機。
6. In case the space provided hereunder is not sufficient, you can request for additional paper sheet/s.
Please ensure you write the appropriate question number corresponding to your answer on the additional sheet/s.
如以下所提供的空白位置不足，請向在場工作人員索取額外紙張，並於適當的位置填寫試題編號。

Seat no. : _____

Applicant Number 考生編號 : AP 14-_____



1. Let α and β be the roots of the equation $10x^2 - 20x + 1 = 0$. Without solving the equation, find the values of

設 α and β 為方程 $10x^2 - 20x + 1 = 0$ 的根。無須解以上方程，求下列兩式的值

A, $4^\alpha \times 4^\beta$ (2 marks 分)

$$\text{Sum of the roots: } \alpha + \beta = -\left(\frac{-20}{10}\right) = 2$$

$$4^\alpha \times 4^\beta = 4^{\alpha+\beta} = 4^2 = 16.$$

B, $\log_{10} \alpha + \log_{10} \beta$ (3 marks 分)

$$\text{Product of roots: } \alpha\beta = \frac{1}{10}$$

$$\log_{10} \alpha + \log_{10} \beta$$

$$= \log_{10} \alpha\beta = \log_{10} \frac{1}{10}$$

$$= \log_{10} \frac{1}{10}$$

$$= \log_{10} 1^{-1}$$

$$= -1.$$

2. In Figure 1, $ABCD$ is a square with $AB = 5$. E is a point on AD such that $DE = 3$. Find x , y and z . Give your answers correct to the nearest degree. (5 marks)

圖 1 中， $ABCD$ 為一正方形且 $AB = 5$ 。 E 是 AD 上的一點使得 $DE = 3$ 。 求 x 、 y 及 z 的值，答案須準確至最接近的度。 (5 分)

$$\tan x = \frac{3}{5}$$

$$x = 30.96^\circ = 31^\circ. \quad (1.5 \text{ marks})$$

$$\tan y = \frac{2}{5}$$

$$y = 21.80^\circ = 22^\circ. \quad (1.5 \text{ marks})$$

$$\angle AEB = 90^\circ - y$$

$$\angle DEC = 90^\circ - x$$

$$\begin{aligned} \therefore z &= 180^\circ - (90^\circ - y) - (90^\circ - x) \\ &= x + y \\ &= 30.96^\circ + 21.8^\circ \\ &= 53^\circ. \quad (2 \text{ marks}) \end{aligned}$$

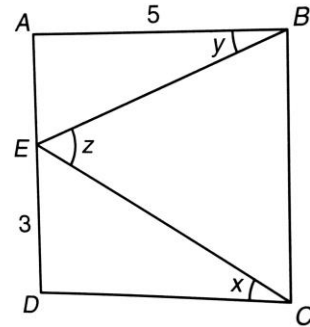


Figure 1 圖 1



3. z varies directly as x^2 and inversely as y . when $x = 4$ and $y = 6$, $z = 8$.
 z 隨 x^2 而正變且隨 y 而反變。當 $x = 4$ 及 $y = 6$ 時， $z = 8$ 。

a, Express z in terms of x and y .
試以 x 及 y 表 z 。

(2 marks)
(2 分)

$$z \propto \frac{x^2}{y}$$

$$\therefore z = \frac{kx^2}{y}$$

By substituting $x = 4$, $y = 6$ and $z = 8$ into

$$z = \frac{kx^2}{y}$$

$$k = 3$$

$$\therefore z = \frac{3x^2}{y}$$

b, If $z = 18x - 27y$, find $x : y : z$.
若 $z = 18x - 27y$, 求 $x : y : z$ 。

(3 marks)
(3 分)

$$18x - 27y = \frac{3x^2}{y}$$

$$18xy - 27y^2 = 3x^2$$

$$x^2 - 6xy + 9y^2 = 0$$

$$(x - 3y)^2 = 0$$

$$x = 3y.$$

$$\therefore z = \frac{3(3y)^2}{y} = 27y.$$

$$\therefore x : y : z = 3y : y : 27y = 3 : 1 : 27.$$



4. A game is played as follows. The player throws a die until he gets a '6'. Every time when the player throws the die and get a number not equal to '6', he can get 1 mark.

一個遊戲的規則如下。參加者不斷投擲一顆骰子，直至得到 '6' 為止。每次當參加者投擲骰子，而所得數字不是 '6' 的話，他便可得 1 分。

a, Find the probability that the player gets
求參加者得到以下分數的概率

(i) 1 mark 分 (1 mark 分)

(ii) 2 marks 分 (1 mark 分)

$$P(1 \text{ mark}) = P(\text{not '6' in the first throw}) \times P(\text{'6' in the second throw})$$

$$= \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}.$$

$$P(2 \text{ marks}) = P(\text{not '6' in the first throw}) \times P(\text{not '6' in the second throw}) \times P(\text{'6' in the third throw})$$

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}.$$

b, Find, in terms of n , the probability that the player gets n marks. (3 marks 分)

求參加者得到 n 分的概率，答案以 n 表示。

$$P(n \text{ marks}) = \left(\frac{5}{6}\right)^n \times \frac{1}{6}$$

$$= \frac{5^n}{6^{n+1}}.$$



5. There are six positive integers. The mode is 2, the median is 4, the largest is 8 and the smallest is 1. What is the mean? (5 marks)

六個正數的眾數是 2、中位數是 4、最大是 8、最小是 1。平均數是甚麼？(5 分)

Let the six integers, in ascending order, be 1, a, b, c, d and 8 respectively.

If $a=1$, then there are two '1' in the distribution and for mode equals to 2, we must have $b=c=d=2$. Then the median $= \frac{b+c}{2} = 2$, and this contradicts the given condition. Therefore $a \neq 1$.

Being mode equals to 2, so a and b must be 2. then the median $4 = \frac{b+c}{2}$, and this gives $c=6$ as $b=2$. Now d can only be the number between 6 to 8 and '6' or '8' has the same frequency as the mode of '2' and this is not allowed. Therefore d must be 7.

$$\text{Hence the mean} = \frac{1+2+2+6+7+8}{6}$$

$$=4.33. \text{ (2 decimal places).}$$

(Part II) Multiple-choice question (第二部份) 選擇題

6. The equation of a circle is $x^2 + y^2 - 4x + 2y + 1 = 0$. Which of the following is/are true?

圓的方程是 $x^2 + y^2 - 4x + 2y + 1 = 0$ 。下列何者必為正確？

- | | |
|---|----------------------|
| I. The centre is $(-2, 1)$. | I. 圓中心是 $(-2, 1)$ 。 |
| II. The radius is 2 units. | II. 半徑為 2。 |
| III. The circle intersects the y-axis at two distinct points. | III. 圓相交於 y 軸兩個不同的點。 |

- | | |
|-------------------|--------------|
| A. I only | 只有 I |
| B. II only | 只有 II |
| C. III only | 只有 III |
| D. I and II only | 只有 I 及 II |
| E. II and III | 只有 II 及 III |

7. In the figure 2, the graph of $y = x^2 + bx + c$ cuts the x-axis at A and B. $OA + OB =$

在圖 2，曲線圖 $y = x^2 + bx + c$ 與 x 軸相交於 A 和 B。 $OA + OB =$

- | |
|---------------------------|
| A. b |
| B. c |
| C. $-b$ |
| D. $-c$ |
| E. $-\frac{b}{c}$ |

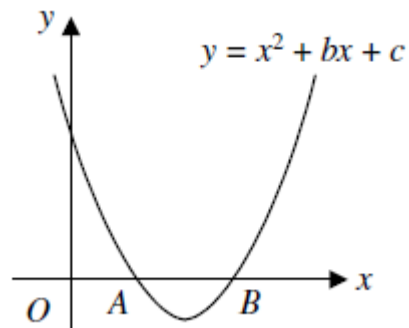


Figure 2. 圖 2。



8. x, y, z are three consecutive positive integers. Which of the following is true?
 x, y, z 是三個連續正整數。下列何者為正確?

- | | |
|---|-------------------------------|
| A. $x + y + z$ must be odd | $x + y + z$ 必須是奇數 |
| B. $x + y + z$ must be even | $x + y + z$ 必須是偶數 |
| C. xyz must be odd | xyz 必須是奇數 |
| D. xyz must be even | xyz 必須是偶數 |
| E. $x^2 + y^2 + z^2$ must be even | $x^2 + y^2 + z^2$ 必須是偶數 |

9. If the solution of the inequality $x^2 - ax + 6 \leq 0$ is $c \leq x \leq 3$, then
若不等式 $x^2 - ax + 6 \leq 0$ 的解為 $c \leq x \leq 3$ ，則

- A. $a = 5, c = 2$**
- B. $a = -5, c = 2$
- C. $a = 5, c = -2$
- D. $a = 1, c = -2$
- E. None of the above 以上皆不是



10. The sum of all the integers between 1 and 300 which are divisible by 7 is
由 1 至 300 中，所有能被 7 整除的整數之和為

- A. 6174
- B. 6321**
- C. 12642
- D. 28000
- E. None of the above 以上皆不是

11. If $x = (p + qy^2)^{\frac{1}{4}}$, then $y =$
若 $x = (p + qy^2)^{\frac{1}{4}}$, 則 $y =$

- A. $\pm \sqrt{\frac{x^2 - p}{q}}$
- B. $\pm \sqrt{\frac{p - x^2}{q}}$
- C. $\pm \sqrt{\frac{p - x^4}{q}}$
- D. $\pm \sqrt{\frac{x^4 - p}{q}}$**

E. None of the above 以上皆不是



12. The price of a diamond ring is increased by 4%, 6% and 8% in three successive years. Find the percentage change of the price of the diamond ring over the past three years.

連續三年間，一顆鑽石戒子的價值分別增加了 4%、6% 及 8%。求它的價值在這三年間之整體百分數變化。

- A. 16%
- B. 18%
- C. 19.1%
- D. 20.1%
- E. None of the above 以上皆不是

13. Suppose $P(x) = x^2 - kx + 6$. If $P(x)$ is divisible by $x+2$, find the remainder when $P(x)$ is divided by $x+1$.

假設 $P(x) = x^2 - kx + 6$ 。若 $P(x)$ 可被 $x+2$ 整除，求 $P(x)$ 除以 $x+1$ 的餘數。

- A. 2
- B. 5
- C. 8
- D. 12
- E. None of the above 以上皆不是

14. In figure 3, $y = 2x + 7$ cuts the graph of $y = x^2 - 4x$ at points P and Q . Find the coordinates of Q .

在圖 3 中， $y = 2x + 7$ 與 $y = x^2 - 4x$ 相交於 P 及 Q 兩點。求 Q 的坐標。

- A. (-1, 5)
- B. (1, 9)
- C. (5, 17)
- D. (7, 21)**
- E. None of the above 以上皆不是

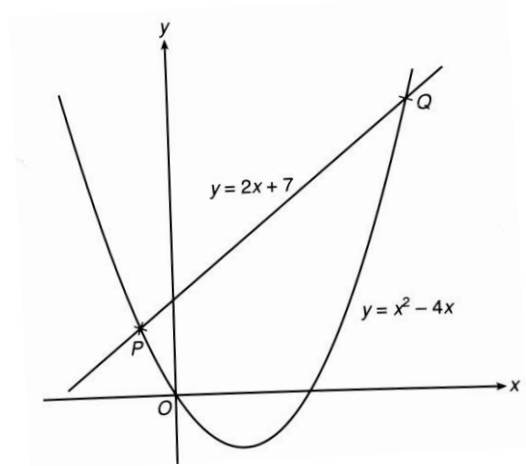


Figure 3. 圖 3。

15. If $x^{\frac{1}{2}} + x^{-\frac{1}{2}} = \sqrt{5}$, then $x + \frac{1}{x} =$

若 $x^{\frac{1}{2}} + x^{-\frac{1}{2}} = \sqrt{5}$ ，則 $x + \frac{1}{x} =$

- A. 1
- B. 3**
- C. 5
- D. 7
- E. None of the above 以上皆不是



16. A bag contains 3 blue balls and 2 green balls. Two balls are drawn randomly from the bag at one time and it is given that one of the drawn ball is blue. Find the probability that the balls drawn are all blue.

一個袋中有 3 個藍色球及 2 個綠色球。現從該袋中隨機取出兩個球，並已知其中一個是藍色的，求所選取的兩個球都是藍色的概率。

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{3}{10}$

D. $\frac{3}{5}$

E. None of the above 以上皆不是

17. Chris sold his car to Mary at a loss of 20%. Mary sold it to Andy at a profit of 10%. If Andy paid \$176,000 for the car, find Chris's loss.

克里斯將一輛車售給瑪麗、虧蝕了 20%。瑪麗將該輛汽車轉售給安迪，獲利 10%。若安迪需付 \$176,000，求克里斯的虧蝕。

A. \$24,000

B. \$40,000

C. \$56,000

D. \$72,000

E. None of the above 以上皆不是



18. If $4^x = a$, then $16^x =$

若 $4^x = a$, 則 $16^x =$

- A. $4a$
- B. a^2
- C. a^4
- D. 2^a
- E. None of the above 以上皆不是

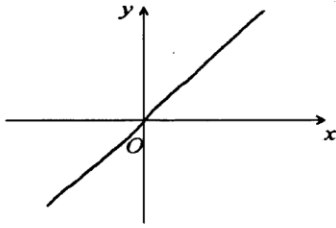
19. $\tan \theta + \frac{1}{\tan \theta} =$

- A. 1
- B. $\frac{2}{\tan^2 \theta}$
- C. $\sin \theta \cos \theta$
- D. $\frac{1}{\sin \theta \cos \theta}$
- E. None of the above 以上皆不是

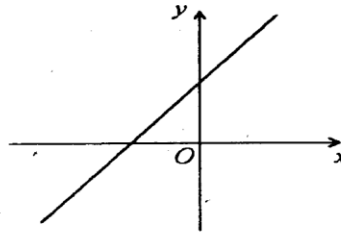
20. If a, b and c are all positive, which of the following may represent the graph of $ax + by + c = 0$?

若 a, b 及 c 均為正數，下列何者可表示 $ax + by + c = 0$ 的圖像

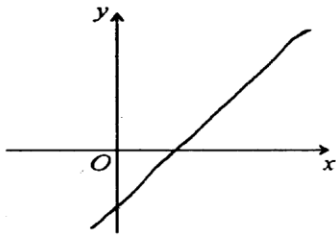
A.



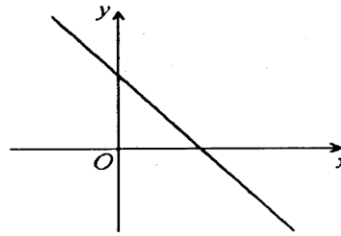
B.



C.



D.



E

