2015/2016 學年入學/分班考試《數學》卷二

Admission/Placement Examination 2015/2016 – MATHEMATICS（Paper 2）

第一部分：選擇題

從四個選項中選擇一個正確答案，每題 5 分，共 50 分

1. 已知 a,b 為實數，則 $|a|>|b|$ 的充分必要條件是（　　　）

A. $\ln a>\ln b$　　B. $a^3>b^3$　　C. $a-b>0$　　D. $a^2>b^2$

2. 已知集合 $A=\{x\mid x^2-3x-4<0\}$，$B=\{x\mid \frac{x-5}{x}\le 0\}$，則 $A\cap B=$ (　　)

A. $(-1,0)$　　B. $(-1,5)$　　C. $(0,4)$　　D. $[0,4)$

3. 記 $f(x)=a\sin(\pi x+\alpha)+b\cos(\pi x+\beta)$, 若 $f(2014)=5$, 則 $f(2015)=$（　　　）

A. -5　　B. 5　　C. 4　　D. -4

4. 已知數列 $\{a_n\}$ 的通項 $a_n=\dfrac{1}{\sqrt{n}+\sqrt{n+1}}(n\ge 1)$，那麼此數列的前 99 項和 $S_{99}=$ （　　　）

A. 8　　B. 9　　C. 10　　D. $\sqrt{101}-1$

5. 設 $f(x)=\ln x^2$, $g(x)=e^x$，則 $g[f(\frac{1}{2})]=$（　　　）

A. $\frac{1}{2}$　　B. $e^{\frac{1}{4}}$　　C. $\frac{1}{4}$　　D. 1

6. 函數 $f(x)=\dfrac{\sqrt{1-x^2}}{1-|x|}$ 的定義域為（　　　）

A. $(-1,1)$　　B. $[-1,1)$　　C. $(-\infty,-1)\cup(1,+\infty)$　　D. $(-\infty,-1]\cup(1,+\infty)$

7. 二次函數 $f(x)=ax^2+bx+c$ 對定義域內的任一個 x，恒有 $f(x)=f(18-x)$，且 $f(7)>f(6)$，則（　　　）

A. $f(13)<f(6)<f(9)$　　B. $f(13)<f(9)<f(6)$

C. $f(6)<f(13)<f(9)$　　D. $f(6)<f(9)<f(13)$

8. 奇函數 $f(x)$ 的定義域為 R， $f(x+2)$ 是偶函數， $f(1)=1$，則 $f(7)+f(8)$ 的值為(　　)

A. -2　　B. -1　　C. 0　　D. 1

9. 一家醫院某天出生了 3 個嬰兒，假設生男生女的機會相同，那麼這 3 個嬰兒中，出現 1 個男嬰、2 個女嬰的概率是（　　　）

A. $\frac{1}{3}$　　B. $\frac{3}{8}$　　C. $\frac{2}{5}$　　D. $\frac{1}{4}$

10. 用數字 1，2，3，4，5 組成的無重複數字的四位偶數的個數為 (　　)

A. 8　　B. 24

C. 48　　　　D. 120

第二部分：計算題

要求寫出必要計算或證明步驟，否則將酌情扣分，每題 10 分，共 50 分

11. 已知函數 $f(x)=1+\sin x-\sqrt{3}\cos x$，求

(1) 最小值；

(2) 最小正週期及單調遞增區間.

12. 已知$1,a_1,43$成等差數列，$-1,b_1,b_2,b_3-9$成等比數列，求

(1) a_1，b_2的值；

(2) 若等差數列$\{T_n\}$以a_1為首項，b_2為公差，求數列$\{T_n\}$的前n項和S_n的最大值.

13. 已知二次函數$y=(k^2-2)x^2-4kx+m$的圖像關於直線$x=2$對稱，且它的最低點在直線$y=-\frac{1}{2}x+2$上，求二次函數的解析式.

14. 已知圓 C 的方程為：$(x-2)^2+(y-1)^2=9$，求過點$M(2,-2)$與圓 C 相切的直線方程.

15. 用數學歸納法證明下述不等式:

$$\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{3n}>\frac{9}{10}(n\in N^*,且n\geq 2).$$

Part 1：Multiple-Choice

Choose the best answer to each question, 5 points each, 50 points total.

1. Let $a,b \in R$, which one of the following is the necessary and sufficient condition of $|a|>|b|$?

( )

A. $\ln a > \ln b$ B. $a^3 > b^3$ C. $a-b>0$ D. $a^2 > b^2$

2. If $A=\{x \mid x^2-3x-4<0\}, B=\{x \mid \frac{x-5}{x} \le 0\}$,then $A \cap B =$ ( )

A. $(-1,0)$ B. $(-1,5)$ C. $(0,4)$ D. $[0,4)$

3. Given that $f(x)=a\sin(\pi x+\alpha)+b\cos(\pi x+\beta)$, $f(2014)=5$, then $f(2015)=$( )

A. -5 B. 5 C. 4 D. -4

4. Given a sequence $\{a_n\}, a_n = \frac{1}{\sqrt{n}+\sqrt{n+1}} (n \ge 1)$, its sum of first 99 terms

$S_{99} =$( )

A. 8 B. 9 C. 10 D. $\sqrt{101}-1$

5. Suppose $f(x)=\ln x^2$, $g(x)=e^x$, then $g[f(\frac{1}{2})]$=( )

A. $\frac{1}{2}$ B. $e^{\frac{1}{4}}$ C. $\frac{1}{4}$ D. 1

6. The domain of $f(x)=\frac{\sqrt{1-x^2}}{1-|x|}$ is ( )

A. $(-1,1)$ B. $[-1,1)$ C. $(-\infty,-1)\cup(1,+\infty)$ D. $(-\infty,-1]\cup(1,+\infty)$

7. A quadratic function $f(x)=ax^2+bx+c$ satisfies $f(x)=f(18-x)$ and

$f(7)>f(6)$, then we can conclude that ( )

A. $f(13)<f(6)<f(9)$ B. $f(13)<f(9)<f(6)$

C. $f(6)<f(13)<f(9)$ D. $f(6)<f(9)<f(13)$

8. The domain of the odd function $f(x)$ is R ,if $f(x+2)$ is an even function, and $f(1)=1$,then the value of $f(7)+f(8)$ is ( )

A. -2 B. -1 C. 0 D. 1

9. Suppose three babies were born at a hospital one day, the same opportunity between boys and girls,

then the probability of just appearing a baby boy and two

baby girls is ( )

A. $\frac{1}{3}$ B. $\frac{3}{8}$ C. $\frac{2}{5}$ D. $\frac{1}{4}$

10. Choose 4 numbers from 1, 2, 3, 4, 5 and compose a four-figure-even number, that does not have the same number. The total number of different choices is ( )

A. 8 B. 24

C. 48 D. 120

Part II：Calculations

Show all your steps or proofs in getting the answers. Full credits will be given only if the answer and all steps are correct and clearly shown, 10 points each, 50 points total.

11. Given $f(x) = 1 + \sin x - \sqrt{3}\cos x$, find

(1) the minimum of $f(x)$;

(2) the least positive period and the increasing interval(s).

12. Soppose that $1, a_1, 43$ is an arithmetic sequence, $-1, b_1, b_2, b_3 - 9$ is a geometric sequence,

(1) find the value of a_1, b_2;

(2) if the arithmetic sequence $\{T_n\}$ satisfies $T_1 = a_1$, $d = b_2$, find the maximum of its sum of first n terms S_n.

13. Given the graph of the quadratic function $y = (k^2 - 2)x^2 - 4kx + m$ is symmetric with respect to $x = 2$, and its lowest point is on the straight line $y = -\frac{1}{2}x + 2$, find an equation of the quadratic function.

14. Find the equation(s) of the tangent line to the graph of $(x-2)^2 + (y-1)^2 = 9$ at the point $M(2, -2)$.

15. Prove the following inequality by mathematical induction.

$$\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{3n} > \frac{9}{10} (n \in N^*, n \geq 2).$$