2015/2016 學年入學/分班考試《數學》卷一

Admission/Placement Examination 2015/2016 – MATHEMATICS（Paper 1）

第一部分：選擇題

從四個選項中選擇一個正確答案，每題 5 分，共 50 分

1. 若集合 $A=\{x|x\le -10 或 x\ge -1\}$, $B=\{x|x<-5\}$, $C=\{x|-6\le x<2\}$，則

   $(A\cup B)\cap C$ 中整數的個數是（　　　）。

   A. 2　　B. 3　　C. 4　　D. 5

2. 若關於 x 的一元二次方程 $(m-2)^2x^2+(2m+1)x+1=0$ 有兩個不相等的實根，則 m 的取值範圍是（　　）

   A. $m<\frac{3}{4}$　　B. $m\le\frac{3}{4}$

   C. $m>\frac{3}{4}$ 且 $m\neq 2$　　D. $m\ge\frac{3}{4}$ 且 $m\neq 2$

3. 若 a,b 是任意實數，且 $a>b$，則（　　）

   A. $a^2>b^2$　　B. $\frac{b}{a}<1$

   C. $\lg(a-b)>0$　　D. $(\frac{1}{2})^a<(\frac{1}{2})^b$

4. 設 $f(x)$ 為定義在 $(-\infty,+\infty)$ 上的偶函數，且 $f(x)$ 在 $[0,+\infty)$ 為增函數，則 $f(-2)$、$f(-\pi)$、$f(3)$ 的大小順序是（　　）

   A. $f(-\pi)>f(3)>f(-2)$　　B. $f(-\pi)>f(-2)>f(3)$

   C. $f(-\pi)<f(3)<f(-2)$　　D. $f(-\pi)<f(-2)<f(3)$

5. 一次函數 $y=-\frac{m}{n}x+\frac{1}{n}$ 的圖象同時經過第一、三、四象限的必要但不充分條件是（　　）

   A. $m>1,n<-1$　　B. $mn<0$

   C. $m>0,n<0$　　D. $m<0,n<0$

6. 若直線 l 經過點 $(a-2,-1)$ 和 $(-a-2,1)$，且與經過點 $(-2,1)$，斜率為 $-\frac{2}{3}$ 的直線垂直，則實數 a 的值是（　　）

   A. $-\frac{2}{3}$　　B. $-\frac{3}{2}$

   C. $\frac{2}{3}$　　D. $\frac{3}{2}$

7. 等差數列 $\{a_n\}$ 中，$a_1+a_2+a_3=-24$，$a_{18}+a_{19}+a_{20}=78$，則此數列前 20 項的和為（　　）

   A. 160　　B. 180

   C. 200　　D. 220

8. 在由數字 1，2，3，4，5 組成的所有沒有重複數字的 5 位數中，大於 23145 且小於 43521 的數共有（　　）個.

A. 56　　B. 57　　C. 58　　D. 60

9. 在 5 件產品中，有 3 件一等品和 2 張二等品，從中任取 2 件，那麼以 $\frac{7}{10}$ 為概率的事件是(　　)

A. 都不是一等品　　B. 恰有一件一等品

C. 至少有一件一等品　　D. 至多一件一等品

10. $(3x+\frac{1}{x\sqrt{x}})^n\ (n\in N^*)$的展開式中含有常數項的最小的 n 為(　　)

A. 4　　B. 5　　C. 6　　D. 7

第二部分：計算題

要求寫出必要計算或證明步驟，否則將酌情扣分，每題 10 分，共 50 分

11. 已知 $\tan(\frac{\pi}{4}+\alpha)=\frac{1}{2}$，(1) 求 $\tan\alpha$ 的值；(2) 求 $\frac{\sin 2\alpha-\cos^2\alpha}{1+\cos 2\alpha}$ 的值.

12. 已知雙曲線與橢圓 $\frac{x^2}{49}+\frac{y^2}{24}=1$ 共焦點，且以 $y=\pm\frac{4}{3}x$ 為漸近線，求雙曲線方程.

13. 在數列$\{a_n\}$中，已知 $a_1=-1, a_n+a_{n+1}+4n+2=0.$

(1) 若 $b_n=a_n+2n$，求證：$\{b_n\}$ 為等比數列，並寫出$\{b_n\}$的通項公式；

(2) 求$\{a_n\}$的通項公式.

14. 已知函數 $f(x)=\lg\frac{1+x}{1-x}$，(1) 求 $f(x)$ 的定義域； (2) 使 $f(x)>0$ 的 x 的取值範圍.

15. 用數學歸納法證明：$9^n-8n-1\ \ (n\in N^*)$能夠被 64 整除.

Part 1：Multiple-Choice

Choose the best answer to each question, 5 points each, 50 points total.

1. If $A=\{x|x\le -10 \text{ or } x\ge -1\}$, $B=\{x|x<-5\}$, $C=\{x|-6\le x<2\}$, then the number of integers in the set $(A\cup B)\cap C$ is ( )

A. 2 B. 3 C. 4 D. 5

2. If the equation $(m-2)^2x^2+(2m+1)x+1=0$ has two different solutions, then the range of m is ( )

A. $m<\frac{3}{4}$ B. $m\le\frac{3}{4}$

C. $m>\frac{3}{4}$ and $m\neq 2$ D. $m\ge\frac{3}{4}$ and $m\neq 2$

3. Assume that a,b are arbitrary real number, and $a>b$. Then ( )

A. $a^2>b^2$ B. $\frac{b}{a}<1$

C. $\lg(a-b)>0$ D. $(\frac{1}{2})^a<(\frac{1}{2})^b$

4. Assume that $f(x)$ is an even function on $(-\infty,+\infty)$ and is increasing on $[0,+\infty)$. Then $f(-2)$, $f(-\pi)$, $f(3)$ satisfy ( )

A. $f(-\pi)>f(3)>f(-2)$ B. $f(-\pi)>f(-2)>f(3)$

C. $f(-\pi)<f(3)<f(-2)$ D. $f(-\pi)<f(-2)<f(3)$

5. ( ) is a necessary not sufficient condition of the proposition, that is, the graph of the linear function $y=-\frac{m}{n}x+\frac{1}{n}$ passes through the first, third and fourth quadrants.

A. $m>1, n<-1$ B. $mn<0$

C. $m>0, n<0$ D. $m<0, n<0$

6. Assume that the line l passes through points $(a-2,-1)$, $(-a-2,1)$ and is perpendicular to the line with the slope $-\frac{2}{3}$ that passes through the point $(-2,1)$. Then the value of a is ( )

A. $-\frac{2}{3}$ B. $-\frac{3}{2}$

C. $\frac{2}{3}$ D. $\frac{3}{2}$

7. In the arithmetic progression $\{a_n\}$，$a_1+a_2+a_3=-24$，$a_{18}+a_{19}+a_{20}=78$，then the sum of the first 20 items of the progression is ( )

A. 160 B. 180

C. 200 D. 220

8. How many 5-digit numbers that consist of number 1, 2, 3, 4, 5 with no repeat, greater than 23145 and

less than 43521? (　　　)

A. 56　　　　　B. 57　　　　　C. 58　　　　　D. 60

9. Assume that there are 3 first-class products and 2 second-class products in the 5 products. Choose 2 products from the 5 products, which event in the following options has the probability $\frac{7}{10}$ (　　)

A. Neither is first-class product

B. Only one of the 2 products is first-class product

C. At least one of the 2 products is first-class product

D. At most one of the 2 products is first-class product

10. The smallest positive integer n which makes the expansion of $(3x+\frac{1}{x\sqrt{x}})^n$ ($n\in N^*$) contains a constant term is (　　　)

A. 4　　　B. 5　　　C. 6　　　D. 7

Part II：Calculations

Show all your steps or proofs in getting the answers. Full credits will be given only if the answer and all steps are correct and clearly shown, 10 points each, 50 points total.

11. Assume that $\tan(\frac{\pi}{4}+\alpha)=\frac{1}{2}$. Then

(1) find the value of $\tan\alpha$ ；(2) find the value of $\frac{\sin 2\alpha-\cos^2\alpha}{1+\cos 2\alpha}$.

12. Assume that a hyperbola has the same focal points with the ellipse $\frac{x^2}{49}+\frac{y^2}{24}=1$ and the asymptotic lines of the hyperbola are $y=\pm\frac{4}{3}x$. Find the equation of the hyperbola.

13. Assume that $a_1=-1, a_n+a_{n+1}+4n+2=0$ in the progression $\{a_n\}$.

(1) If $b_n=a_n+2n$, then prove that $\{b_n\}$ is a geometric progression, and find the general formula of $\{b_n\}$ ；

(2) Find the general formula of $\{a_n\}$.

14. Let $f(x)=\lg\frac{1+x}{1-x}$. Then

(1) find the domain of $f(x)$ ； (2) find the range of x such that $f(x)>0$.

15. Prove the following conclusion by mathematical induction.

9^n-8n-1（$n\in N*$） can be divided by 64.