旅 遊 學 院
INSTITUTO DE FORMAÇÃO TURÍSTICA
Institute for Tourism Studies

Admission Examination 2015/2016
2015/2016 年度入學考試
Mathematics 數科
21/03/2015

Time allowed: Two hours **Total Marks: 100**
考試時間：二小時 **總分: 100**

1. This examination consists of two parts: (Part I) 5 work problems and (Part II) 15 multiple-choice questions (a total of 14 pages). Answer all of them.
此份試卷共有兩部份: (第一部份) 5 題答題 和 (第二部份) 15 題選擇題 (共14頁)，請全部作答。

2. (Part I) In order to obtain full credit in each question, you have to show all the steps in your calculations leading to a correct answer in the space provided in this booklet.
(第一部份) 如想取得滿分，必須於此試卷內每個問題下所提供之空白位置詳細列明計算步驟。

3. (Part II) You are required to record your answer by clearly circle **one and only one** of the five alternatives A, B, C, D or E that corresponds to your solution
(第二部份) 每一題所列出的五個答案中，只有一個答案是正確的，請選擇**圈出一個**正確的答案。

4. (Part II) The grading scheme will be as follows:
(第二部份) 評分標準如下:

Correct answer: +5 points , Incorrect answer: 0 point , No response: 0 point.
正確答案：加 5 分， 錯誤答案: 零分， 不作答：零分。

5. You may use a non-programmable calculator
可使用沒有設定方程式的計算機。

6. In case the space provided hereunder is not sufficient, you can request for additional paper sheet/s. Please ensure you write the appropriate question number corresponding to your answer on the additional sheet/s.
如以下所提供的空白位置不足，請向在場工作人員索取額外紙張，並於適當的位置填寫試題編號。

Seat no. : _____

Applicant Number 考生編號 : AP 15-_____

1. In an IQ competition, the mean score of a team of *m* men and *n* women is 70.
在一智力比賽中，某隊有男子 *m* 人和女子 *n* 人，而全隊的平均積分為 70。

a. Find the total score of the team in terms of *m* and *n*. (1 mark)
求全隊的總積分（答案以 *m*、*n* 表示）。 (1 分)

$$\frac{\sum x_i}{m+n} = 70$$

Total score = 70 (*m*+*n*).

b. If the mean score of the men is 75 and the mean score of the women is 62, find the ratio *m* : *n*. (2 marks)
若男子的平均積分為 75，女子的平均積分為 62，求 *m* : *n*。 (2 分)

$70(m+n) = 75m + 62n$

$5m = 8n$

$$\frac{m}{n} = \frac{8}{5}$$

m : *n* = 8 : 5

c. If there are altogether 39 persons in the team, find the number of men. (2 marks)
若全隊共有 39 人，求男子的人數。 (2 分)

$$\text{Number of men} = 39 \times \frac{8}{8+5} = 24.$$

2. Solve the following equations without using a calculator:
解下列方程（考生不可使用計算機）

a. $3^x = \dfrac{1}{\sqrt{27}}$ (2 marks 分)

$$3^x = \frac{1}{(3^3)^{\frac{1}{2}}} = \frac{1}{3^{\frac{3}{2}}}$$

$$3^x = 3^{-\frac{3}{2}}$$

$$x = -\frac{3}{2}.$$

b. $\log x + 2 \log 4 = \log 48$ (3 marks 分)

$$\log x + \log 4^2 = \log 48$$

$$\log 16x = \log 48$$

$$16x = 48$$

$$x = 3.$$

3. Consider the sequence 3, 9, 27, 81,... . If the sum of the first 2*n* terms is 10 times the sum of the first *n* terms, find the possible value(s) of *n*. (5 marks)
考慮數列 3、9、27、 81、… 。若首 2n 項之和是首 *n* 項之和的 10 倍，求 *n* 的可能值。 (5 分)

The sum of the first n terms = $\frac{3(3^n-1)}{3-1}=\frac{3}{2}(3^n-1)$.

The sum of the first 2n terms = $\frac{3(3^{2n}-1)}{3-1}=\frac{3}{2}(3^{2n}-1)$

$$\frac{3}{2}(3^{2n}-1)=10\cdot\frac{3}{2}(3^n-1)$$

$$(3^{2n}-1)=10\cdot(3^n-1)$$

$$(3^n)^2-10\cdot3^n+9=0$$

$$(3^n-1)\cdot(3^n-9)=0$$

$$3^n=1 \text{ or } 9$$

n= 0 (rejected) or 2.

4. In a mathematics test, there were three multiple choice questions, each with 5 options. Mary sat for the test, but she had only studies 70% of the tested topics, so the probability that she knew how to do a question was 70%. If she knew how to do a question, the probability that she made a careless mistake was 10%. If she didn't know how to do a question, she would guess an answer randomly.
某次數學測驗有三道選擇題，每題有五個選擇。瑪麗在這次測驗中，只溫習了測驗範圍的 70%，所以她懂得回答問題的概率是 70%。若她懂得回答，她大意犯錯的概率是 10%。若她不懂得回答，她會隨意猜一個答案。

a. Find the probability that
求以下各項的概率

i) Mary didn't know how to do it and guessed a wrong answer. (1 mark)
瑪麗不懂得回答並且猜錯答案。 (1 分)

$$(1-0.7)\times\frac{4}{5}=0.24.$$

ii) Mary answered it correctly. (1 mark)
瑪麗答對。 (1 分)

$$1-P(\text{she answered wrongly})$$
$$=1-(0.07+0.24)$$
$$=0.69.$$

b. To pass the test, a student must answer at least two questions correctly. What is the probability that Mary failed in the test? (3 marks)
學生必須在測驗中答對最少兩題，才可以合格。求瑪麗在這次測驗中不合格的概率。 (3 分)

P(She failed in the test) = P(only 1 question is correct) + P(all questions are wrong)

$$=[0.69\times(1-0.69)\times(1-0.69)]\times 3+(1-0.69)^3$$

$$=0.229.$$

5. In figure 1, ΔABD is a right-angled triangle where $\angle ABD = 90^o$ and $\angle ADB = 40^o$. C and E are points on BD and AD respectively such that BC= 2 cm, CE = 4cm and AE= 5cm. Find AB. (5 marks)

圖 1 中，ΔABD 是一個直角三角形，其中 $\angle ABD = 90^o$ 及 $\angle ADB = 40^o$。C 及 E 分別是 BD 及 AD 上的兩點，使得 BC = 2 cm，CE = 4cm 及 AE= 5cm。求 AB。(5 分)

Construct a point F on AB such that $EF \perp AB$.

$\angle BAE = 180^o - 90^o - 40^o$
$= 50^o$

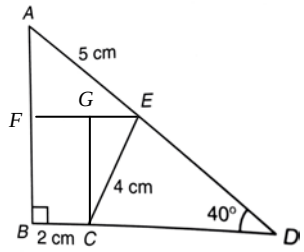

Figure 1. 圖 1。

In ΔAFE,

$$\cos 50^o = \frac{AF}{5\text{cm}}$$

$$AF = 5\cos 50^o \text{ cm}.$$

$$\sin 50^o = \frac{EF}{5\text{cm}}$$

$$EF = 5\sin 50^o \text{ cm}.$$

Construct a point G on EF such that $CG \perp EF$.

$$EG = (5\sin 50^o - 2)\text{ cm}.$$

In ΔCEG,

$$CG = \sqrt{4^2 - (5\sin 50^o - 2)^2} \text{ cm}$$
$$= 3.5567\text{cm}.$$

$$AB = AF + BF$$
$$= AF + CG$$
$$= (5\cos 50^o + 3.5567)\text{ cm}$$
$$= 6.77\text{cm}.$$

(Part II) Multiple-choice question (第二部份) 選擇題

6. There are four number cards:
以下共有四張數字咭:

1 , 2 , 4, 7

John draws two number cards at random. What is the probability that the sum of the two cards is greater than 5?
约翰隨機抽出兩張數字咭，求兩張咭上的數字之和大於 5 的概率。

A. $\frac{1}{2}$
B. $\frac{1}{3}$
C. $\frac{2}{3}$
D. $\frac{5}{6}$
E. None of the above 以上皆不是

7. Simplify (化簡) $\frac{\left(\frac{y}{x}-1\right)\left(1-\frac{x}{y}\right)}{\frac{x}{y}-\frac{y}{x}}$.

A. $\frac{x-y}{x+y}$
B. $-\frac{x-y}{x+y}$
C. $\frac{x+y}{x-y}$
D. $-\frac{x+y}{x-y}$
E. None of the above 以上皆不是.

8. If α, β are the roots of the equation $x^2-4x-3=0$, then $\alpha^2+\alpha\beta+\beta^2=$
若方程$x^2-4x-3=0$的根為 α及β，則$\alpha^2+\alpha\beta+\beta^2=$

A. -13
B. 5
C. 13
D. 16
E. 19

9. *a*, *b*, *c*, *d* are 4 consecutive terms of a geometric sequence. Which of the following must be true?
a、*b*、*c*、*d* 為某等比數列的 4 個連續項。下列何者必為正確？

I. $b^2=ac$
II. $\frac{b}{a}=\frac{d}{c}$
III. $\frac{d}{a}=\left(\frac{c}{b}\right)^3$

A. II only 只有 II
B. I and II only 只有 I 及 II
C. I and III only 只有 I 及 III
D. II and III 只有 II 及 III
E. I, II and III 只有 I, II 及 III

10. In a class, students study either History or Geography, but not both. If the number of students studying Geography is 50% more than those studying History, what is the percentage of students studying History?
某班的學生修讀歷史或地理，但不可同時修讀這兩科。若修讀地理的學生比修讀歷史的學生多 50%，求修讀歷史的學生的百分比。

A. 25%

B. $33\frac{1}{3}\%$

C. 40%

D. 60%

E. $66\frac{2}{3}\%$

11. Peter goes to school and returns home at speed x km/h and $(x+1)$ km/h respectively. The school is 2 km from Peter's home and the total time for the two journeys is 54 minutes. Which of the following equations can be used to find x?
彼得步行上學和返家的速度分別是 x km/h 及 $(x+1)$ km/h。學校和家的距離為 2 km，而往返兩地共需時 54 分鐘。下列哪個方程可用求 x？

A. $\frac{x}{2}+\frac{x+1}{2}=\frac{54}{60}$

B. $\frac{2}{x}+\frac{2}{x+1}=\frac{54}{60}$

C. $\frac{\frac{1}{2}[x+(x+1)]}{4}=\frac{54}{60}$

D. $\frac{4}{\frac{1}{2}[x+(x+1)]}=\frac{54}{60}$

E. None of the above 以上皆不是.

12. If $a : b = c : d = 3 : 1$, then which of the following must be true?
若 $a : b = c : d = 3 : 1$，下列何者必為正確?

A. $a = c = 3$
B. $a =$ c and 及 $b = d$
C. $a + b = c + d$
D. $a + c : b + d = 3 : 1$
E. None of the above 以上皆不是.

13. If $-\frac{1}{2} < x < 4$ and $-\frac{1}{2} < y < 2$, then
若 $-\frac{1}{2} < x < 4$ 及 $-\frac{1}{2} < y < 2$, 則

A. $\frac{1}{4} < xy < 8$
B. $-\frac{1}{4} < xy < 8$
C. $-1 < xy < 8$
D. $-2 < xy < 8$
E. None of the above 以上皆不是.

14. The straight line L: $3x + by + 9 = 0$ intersects the y-axis at (0 , -3). Find the point of intersection of L with the line $4x - 2y - 7 = 0$.

直線 L: $3x + by + 9 = 0$ 與 y 軸相交於 (0 , -3) 。求直線 L 與直線 $4x - 2y - 7 = 0$ 的交點。

A. $-\frac{1}{4}, -\frac{19}{4}$

B. $\frac{1}{4}, -\frac{19}{4}$

C. $\frac{1}{6}, -\frac{19}{6}$

D. $-\frac{1}{6}, -\frac{19}{6}$

E. None of the above 以上皆不是.

15. Suppose $Q(x) = x^3 + kx^2 + 2kx + 8$ and $Q(4) = 0$. Factorize $Q(x)$.

已知 $Q(x) = x^3 + kx^2 + 2kx + 8$ and $Q(4) = 0$.因式分解 $Q(x)$.

A. $(x+1)(x-2)(x-4)$

B. $(x-1)(x+2)(x-4)$

C. $(x-1)(x-2)(x+4)$

D. $(x+1)(x+2)(x-4)$

E. None of the above 以上皆不是.

16. If A, B and C are the angles of a triangle, then $\cos\left(\frac{A+B}{2}\right)=$

若 A、B 及 C 分別為三角形的角，則 $\cos\left(\frac{A+B}{2}\right)=$

A. $\sin\frac{C}{2}$
B. $\cos\frac{C}{2}$
C. $\sin C$
D. $\cos C$
E. None of the above 以上皆不是.

17. Which of the following circles touches the y-axis?
下列哪個圖形與 y 軸相切

A. $(x+4)^2+(y-3)^2=9$
B. $(x-4)^2+(y+3)^2=9$
C. $(x+4)^2+(y-3)^2=16$
D. $(x-4)^2+(y-3)^2=36$
E. None of the above 以上皆不是.

18. If n is an even number, which of the following must be an odd number?
若 n 為偶數，則下列何者必為奇數?

I. $\frac{n+1}{2}$

II. $\frac{n}{2}$

III. $(n+1)(n-1)$

A. I only 只有 I
B. II only 只有 II
C. III only 只有 III
D. I and III only 只有 I 及 III
E. I, II and III I 、II 及 III

19. If $\log(2y-1) = \log(y+2) - 1$, then y =
若 $\log(2y-1) = \log(y+2) - 1$ ，則 y =

A. $\frac{1}{2}$
B. $\frac{6}{7}$
C. $\frac{12}{19}$
D. $\frac{29}{19}$
E. None of the above 以上皆不是.

20. The price of gasoline is increased by 25% and a driver reduced his consumption by 30%. The percentage change in his expenditure on gasoline is

汽油的價格上升了 25%，某司機就減少了 30%的用量，則他在汽油上的花費

A. decreased by 12.5%. 減少了 12.5% 。
B. decreased by 5% . 減少了 5% 。
C. increased by 5% . 增加了 5% 。
D. increased by 7.5%. 增加了 7.5%。
E. None of the above 以上皆不是.