INSTITUTE FOR TOURISM STUDIES
旅遊學院
Admission Examination 2013/2014
2013/2014 年度入學考試
Mathematics 數科
24/03/2013

Time allowed: Two hours **Total Marks: 100**
考試時間：二小時 **總分: 100**

1. This examination consists of two parts: (Part I) 5 work problems and (Part II) 15 multiple-choice questions (a total of 14 pages). Answer all of them.
   此份試卷共有兩部份: (第一部份) 5 題答題 和 (第二部份) 15 題選擇題 (共14頁)，請全部作答。

2. (Part I) In order to obtain full credit in each question, you have to show all the steps in your calculations leading to a correct answer in the space provided in this booklet.
   (第一部份) 如想取得滿分，必須於此試卷內每個問題下所提供之空白位置詳細列明計算步驟。

3. (Part II) You are required to record your answer by clearly circle **one and only one** of the five alternatives A, B, C, D or E that corresponds to your solution
   (第二部份) 每一題所列出的五個答案中，只有一個答案是正確的，請選擇**圈出一個**正確的答案。

4. (Part II) The grading scheme will be as follows:
   (第二部份) 評分標準如下:

   Correct answer: +5 points , Incorrect answer: 0 point , No response: 0 point.
   正確答案：加 5 分， 錯誤答案: 零分， 不作答：零分。

5. You may use a non-programmable calculator
   可使用沒有設定方程式的計算機。

6. In case the space provided hereunder is not sufficient, you can request for additional paper sheet/s. Please ensure you write the appropriate question number corresponding to your answer on the additional sheet/s.
   如以下所提供的空白位置不足，請向在場工作人員索取額外紙張，並於適當的位置填寫試題編號。

Applicant Number 考生編號 : AP 12-_____

1. Mary lent $129 to Peter. The next day Peter returned $64 to Mary, and on each subsequent day Peter returned half the amount he returned to Mary the previous day, i.e. on the second day he returned $32, and then $16, and so on.
瑪麗借了$129給彼得。翌日，彼得把 $64 還給瑪麗。往後的每一天，彼得把之前一天所還之款項的一半還給瑪麗，即兩天後還款$32，三天後還款$16，如此類推。

A, After ten days, how much did Peter still owe Mary? (3 marks)
還款十天後，彼得還欠瑪麗多少錢? (3 分)

Amount that Peter had returned after ten days = $64+$32+$16……

$$= \left[\frac{64\left(1-\frac{1}{2}\right)^{10}}{1-\frac{1}{2}} \right]$$

=$127.875.

The amount that Peter still owned Mary after ten days = $129 - $127.875
=$1.125.

B, Can Peter repay the debt ever? Explain. (2 marks)
彼得最後能否還清所有欠款? 試解釋。 (2 分)

The maximum amount that Peter return is the sum to infinity of the geometric sequence

$$= \frac{a}{1-R} = \frac{64}{1-\frac{1}{2}}$$

= $128 < $129.

Peter can never repay the debt.

2. Consider the function $f(x) = x^2 + bx - 15$, where b is a constant. It is given that the graph of y = f(x) passes through the point (4 , 9).
考慮函數 $f(x) = x^2 + bx - 15$ ，其中 b 為一常數。已知 y = f(x) 的圖像通過點 (4 , 9)。

A, Find b. Hence, or otherwise, find the two x-intercepts of the graph of y = f(x). (2 marks)
求 b 。由此，或利用其他方法，求 y = f(x) 的圖像的兩個 x 軸截距。(2 分)

$f(4) = 9$

$4^2 + 4b - 15 = 9$

$4b = 8$

$b = 2.$

$f(x) = 0$

$x^2 + 2x - 15 = 0$

$(x - 3)(x + 5) = 0$

$x=3$ or $x=-5$

Thus the two x-intercepts are 3 and -5

B, Let k be a constant. If the equation f(x) = k has two distinct real roots, find the range of values of k. (3 marks)
設 k 為一常數。若方程 f(x) = k 有兩個相異的實根，求 k 的取值範圍。(3 分)

$f(x) = k$

$x^2 + 2x - 15 = k$

$x^2 + 2x - (15 + k) = 0$

$\Delta = 2^2 - 4(1)(-15 - k)$

$=64+4k.$

Since f(x) = k has two distinct real roots,

$64 + 4k > 0$

$k > -16.$

3. Factorize
因式分解

A, $4a^2-9-4ab+6b$ (2 marks 分)

B, $2x^2-8y^2-3x+6y$ (3 marks 分)

A, $4a^2-9-4ab+6b$

$=(4a^2-9)-b(4a-6)$

$=(2a-3)(2a+3)-2b(2a-3)$

$=(2a-3)(2a-2b+3).$

B, $2x^2-8y^2-3x+6y$

$=2(x+2y)(x-2y)-3(x-2y)$

$=(x-2y)[2(x+2y)-3]$

$=(x-2y)(2x+4y-3).$

4. When two fair dice are thrown, what is the probability that
投擲兩顆公平的骰子，求以下各項的概率

A, the sum of the two numbers is 9? (2 marks)
兩數之和是 9? (2 分)

The favorable outcomes are (3 , 6), (4 , 5), (5 , 4) and (6 , 3).

P (sum of the two numbers is 9) = $\frac{4}{36}$

$= \frac{1}{9}.$

B, the product of the two numbers is odd? (3 marks)
兩數之積是奇數? (3 分)

The product is odd when both numbers are odd.

P (Product is odd) = P (first number is odd and second number is odd)

$= \frac{3}{6} \times \frac{3}{6}$

$= \frac{1}{4}.$

5. The sum of two consecutive even integers is greater than 10 and smaller than 16. Find the possible value(s) of the larger number.
兩個連續偶整數的和大於 10 及小於 16。求較大的數的可取值範圍。

Let the smaller even integer be x and the larger one be x+2.

$$10 < x + (x+2) < 16$$

$$10 < 2x+2 < 16$$

$$8 < 2x < 14$$

$$4 < x < 7$$

The possible value of x is 6.

Thus the possible value of the larger number is 8.

(Part II) Multiple-choice question (第二部份) 選擇題

6. If $\frac{a}{b}=\frac{c}{d}\neq 0$, which of the following must be true?

若 $\frac{a}{b}=\frac{c}{d}\neq 0$ ，則下列何者必為正確?

I. $\frac{a}{c}=\frac{b}{d}$

II. $\frac{a+b}{b}=\frac{c+d}{d}$

III. $\frac{a-b}{b}=\frac{c-d}{d}$

A. I only 只有 I
B. III only 只有 III
C. I and II only 只有 I 及 II
D. I and III only 只有 I 及 III
E. I, II and III I 、II 及 III

7. A bag contains 2 black balls and 3 white balls. A boy randomly draws balls from the bag one at a time (without replacement) until a white ball appears. Find the probability that he will make at least 2 draws.
一袋中有 2 個黑球、3 個白球。某男孩從袋中隨機地每次抽取一個球 (取出後並不放回袋中) ，直至抽得白球為止。求他最少要抽 2 次的概率。

A. $\frac{2}{5}$
B. $\frac{3}{5}$
C. $\frac{1}{10}$
D. $\frac{3}{10}$
E. None of the above 以上皆不是

8. A man bought a box of 200 apples for $500. 10 of the apples were rotten and the rest were sold at $4 each. Find his percentage profit correct to 2 significant figures.
某人用 $500 購一箱共 200 個的蘋果，其中 10 個變壞，餘下的以每個$4 售出。求他的賺率，答案須準確至二位有效數字。

A. 34%
B. 38%
C. 52%
D. 57%
E. None of the above 以上皆不是

9. $\frac{1-x}{x^2+4x-5}+\frac{x-1}{x+1}=$

A. $\frac{x^2+3x-6}{(x+1)(x+5)}$
B. $\frac{x^2+5x-4}{(x+1)(x+5)}$
C. $\frac{(x+4)(x-1)}{(x+1)(x+5)}$
D. $\frac{(x-1)(x-4)}{(x+1)(x-5)}$
E. None of the above 以上皆不是

10. In the figure 1, DAB is a straight line. $\tan\theta =$
圖 1 中，DAB 為一直線。$\tan\theta =$

A. $\dfrac{1}{2\tan 20^\circ}$

B. $\dfrac{1}{2}\tan 20^\circ$

C. $\dfrac{2}{\tan 20^\circ}$

D. $2\tan 20^\circ$

E. None of the above 以上皆不是.

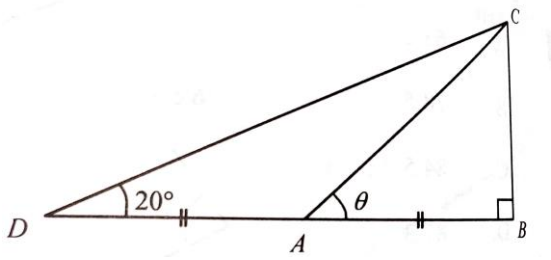

Figure 1 (圖 1)

11. Simplify (化簡) $\dfrac{\log x^3 - \log\dfrac{x}{y}}{\log x^4 y^2}$, where (其中) x , y > 0.

A. 10

B. xy

C. $\dfrac{1}{2}$

D. $\dfrac{x}{y}$

E. None of the above 以上皆不是.

12. The figure 2 shows the graph of $y = ax^2 + bx + c$. Which of the following is true?
圖 2 中所示為 $y = ax^2 + bx + c$ 的圖像。下列何者為正確?

A. $a > 0$, $c > 0$ and 及 $b^2 - 4ac > 0$
B. $a > 0$, $c > 0$ and 及 $b^2 - 4ac < 0$
C. $a > 0$, $c < 0$ and 及 $b^2 - 4ac < 0$
D. $a < 0$, $c > 0$ and 及 $b^2 - 4ac > 0$
E. None of the above 以上皆不是.

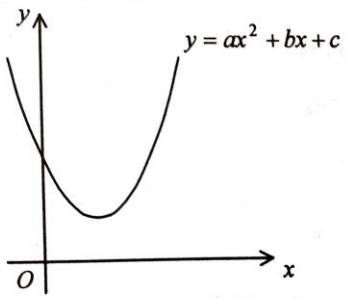

Figure 2 (圖 2)

13. Solve the inequalities (解不等式) $2x + 3 \le x + 1 \le 3x - 5$

A. $x \le -2$
B. $-2 \le x \le 3$
C. $3 \le x \le -2$
D. No solution 無解
E. None of the above 以上皆不是.

14. Let A = (0 , 4) and B = (6 , 0). The equation of the circle of AB as diameter is
設 A = (0 , 4) 、B = (6 , 0)。以 AB 為直徑的圓的方程是

A. $x^2 + y^2 - 4x - 6y = 0$
B. $x^2 + y^2 - 6x - 4y = 0$
C. $x^2 + y^2 - 6x - 4y + 39 = 0$
D. $x^2 + y^2 - 12x - 8y = 0$
E. None of the above 以上皆不是

15. In a group of numbers, if the smallest number is increased by 20, then the mean will be increased by 2. Which of the following must be true?
若將一組數字中最小的一個數增加 20，則該組數字的平均數便增加了 2。下列何者必為正確？

I. The group has 10 numbers.	I. 該組共有 10 個數字。
II. The median remains unchanged.	II. 該組數字的中位數沒有改變。
III. The mode remains unchanged.	III. 該組數字的眾數沒有改變。

A.	I only	只有 I
B.	II only	只有 II
C.	I and II only	只有 I 及 II
D.	I and III only	只有 I 及 III
E.	I, II and III	I 、II 及 III

16. If α and β are roots of the equation $x^2 - 2x - 7 = 0$, then $(2^{\alpha+1})(2^{\beta+1}) =$
若α 和 β 是方程 $x^2 - 2x - 7 = 0$ 的根，則 $(2^{\alpha+1})(2^{\beta+1}) =$

A. 1
B. 8
C. 12
D. 16
E. None of the above 以上皆不是.

17. 60% of the books in a library are new books. If 600 old books are replaced by 600 new books, the amount of new books will increase to 75% of the total. The total number of books in the library is
圖書館內有 60% 的書是新書，當以 600 本新書取代了 600 本舊書後，新書所佔的百分率上升至 75%。那麼圖書館內總共有多少本書？

A. 2800
B. 3000
C. 3600
D. 4000
E. None of the above 以上皆不是.

18. A bag contains three $5 coins and some $2 coins. If the total value of the coins is less than $70, find the greatest possible number of $2 coins.
袋中共有三個 $ 5 硬幣及一些 $ 2 硬幣。若所有硬幣加起來的面值少於 $ 70，則最多共有多少個 $ 2 硬幣？

A. 26
B. 27
C. 27.5
D. 28
E. None of the above 以上皆不是.

19. When a polynomial P(x) is divided by 3x - 4, the remainder is R. What is the remainder when P(x) is divided by 4 - 3x?
當多項式 P(x) 除以 3x - 4 時，其餘數為 R。求當 P(x) 除以 4 - 3x 時的餘數？

A. $-\frac{3}{4}R$
B. $-\frac{4}{3}R$
C. $-R$
D. R
E. None of the above 以上皆不是

20. Which of the following is **not** a geometric sequence?
則下列何者**不是**等比數列?

I. 0.3, 0.33, 0.333, 0.3333, …..
II. 1, 1, 1, 1……..
III. 12, -6, 3, -1.5, …….

A.	I only	只有 I
B.	II only	只有 II
C.	I and II only	只有 I 及 II
D.	I and III only	只有 I 及 III
E.	I, II and III	I 、II 及 III