

2015/2016 學年 入學考試試題 ADMISSION EXAMINATION PAPER

Part 1

1. (a) (4 marks) If $a : b : c = 2 : 5 : 3$, find $\frac{1}{a} : \frac{1}{b} : \frac{1}{c}$.

(4 分) 若 $a : b : c = 2 : 5 : 3$ ，求 $\frac{1}{a} : \frac{1}{b} : \frac{1}{c}$ 。

(b) (4 marks) Solve $\sin \theta = -\cos 20^\circ$, where $0^\circ \leq \theta \leq 360^\circ$.

(4 分) 解 $\sin \theta = -\cos 20^\circ$ ，其中 $0^\circ \leq \theta \leq 360^\circ$ 。

2. In the figure, A and B are points on the x -axis and y -axis respectively. The equations of AC and BC are

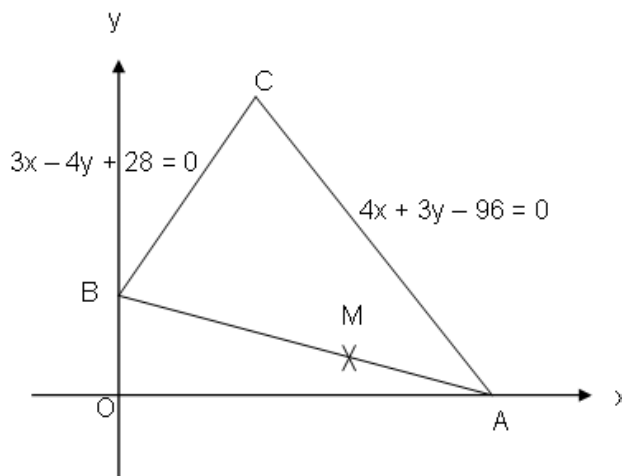
$$4x + 3y - 96 = 0 \text{ and}$$

$$3x - 4y + 28 = 0, \text{ respectively.}$$

M is a point on AB such that $AM = 10$.

在圖中， A 和 B 分別是 x 軸和 y 軸上的點， AC 和 BC 兩條直線的方程分別為 $4x + 3y - 96 = 0$ 和

$3x - 4y + 28 = 0$ 。 M 是 AB 上的一點使 $AM = 10$ 。



- (a) (3 marks) Find the equation of a straight line passing through point C and perpendicular to AB .

(3 分) 求通過 C 點且垂直於 AB 的直線之方程。

- (b) (5 marks) Suppose the straight line passing through the point M and perpendicular to AB intersects AC at N . Prove that $MN = CN$.

(5 分) 設通過 M 點且垂直於 AB 的直線與 AC 相交於 N ，證明 $MN = CN$ 。

3. A sushi restaurant charges its customers as follows:
- I. Any customer who has taken no more than 6 pieces of sushi will pay a fixed basic charge.
 - II. A supplementary fee will be charged on top of the basic charge mentioned in I. This supplementary fee varies directly as the number of additional pieces of sushi taken beyond the first six pieces.

A , B and C went to that sushi restaurant. B and C had taken 8 and 11 pieces of sushi respectively. They had to pay \$128 and \$185 respectively.

某迴轉壽司店的收費方法如下:

- I. 每位進食不多於 6 件壽司的顧客祇需要支付劃一的最低消費。
- II. 除了要支付 I 提到的最低消費外,亦會收取附加費。此附加費隨(6 件以外)額外進食的壽司數目而正變。

A 、 B 及 C 一起光顧該壽司店。 B 和 C 分別吃了 8 件和 11 件壽司,結賬時兩人分別支付\$128 和\$185。

- (a) (5 marks) Let the total charge for a customer taking S pieces of sushi be $\$P$. Establish a formula to calculate the total charge P in terms of S in that restaurant for each of the following two situations.
- (5 分) 設每位惠顧 S 件壽司的顧客的總收費是 $\$P$ 。試就以下兩個情況分別建立一條公式表示該店的總收費 P :
- (i) $S \leq 6$
 - (ii) $S > 6$
- (b) (3 marks) If A 's bill was greater than the total amount that B and C paid together, find the minimum number of pieces of sushi that A had eaten.
- (3 分) 若 A 結賬時須繳付的金額比 B 和 C 加起來所付的總金額還要多,求 A 最少吃了多少件壽司。

4. Let $f(x) = x^3 - 2x^2 - 7x + 2$.
設 $f(x) = x^3 - 2x^2 - 7x + 2$ 。

When $f(x)$ is divided by $x - 2$, the quotient is $Q(x)$ and the remainder is R .

當 $f(x)$ 除以 $x - 2$ 時,商是 $Q(x)$,而餘數是 R 。

- (a) (2 marks) Find $Q(x)$ and R .
(2 分) 求 $Q(x)$ 和 R 。

(b) (3 marks) If $Q(x) \equiv A(x-2)^2 + B(x-2) + C$, find the values of A , B and C .
 (3 分) 若 $Q(x) \equiv A(x-2)^2 + B(x-2) + C$, 求 A 、 B 和 C 的值。

(c) (3 marks) Using the above results, or otherwise, express $f(x)$ in the form
 $f(x) = S(x-2)^3 + T(x-2)^2 + U(x-2) + V$, where S , T , U , V are constants.
 (3 分) 利用以上結果, 或用其他方法, 把 $f(x)$ 寫成
 $f(x) = S(x-2)^3 + T(x-2)^2 + U(x-2) + V$ 的形式, 其中 S 、 T 、 U 、 V 為常數。

5. Bag A contains four banknotes of face values \$10, \$20, \$50 and \$100 respectively. Bag B contains five banknotes of face values \$20, \$50, \$100, \$500 and \$1000 respectively. A banknote is drawn randomly from each of bag A and bag B separately. Assume that in each bag, each banknote has equal chance to be drawn.

袋 A 內有四張鈔票, 面值分別為\$10、\$20、\$50 和 \$100; 袋 B 內有五張鈔票, 面值分別為\$20、\$50、\$100、\$500 和 \$1000。現分別從袋 A 和袋 B 各隨意抽出一張鈔票。假設在每個袋中每張鈔票被抽中的概率相等。

(a) (4 marks) Find the probability that the total amount of the face values of the two banknotes drawn is divisible by 3.

(4 分) 求抽出的兩張鈔票之總面值可被 3 整除的概率。

(b) (4 marks) Given that the total amount of the face values of the two banknotes drawn is more than \$100 but less than \$500, what is the probability that the face value of the banknote drawn out from bag B is \$50?

(4 分) 已知抽出的兩張鈔票之總面值是多於\$100 但少於\$500, 求從袋 B 抽出的鈔票之面值是\$50 的概率。

Part 2

1. If “ $x \propto y$ ” represents “ x is directly proportional to y ”, which of the following is/are true?

若 “ $x \propto y$ ” 表示 “ x 與 y 成正比”，以下哪項是正確的？

I. $y \propto x$ II. $x^2 \propto y^2$ III. $x^2 \propto xy$

- A. I only 只有 I
 B. I and II only 只有 I 和 II
 C. I and III only 只有 I 和 III
 D. II and III only 只有 II 和 III
 E. I, II and III I、II 和 III

2. If $\cos \theta \neq 0$, then $\frac{\cos^2 \theta + \tan^2 \theta - 1}{\sin^2 \theta + \cos^2 \theta - \tan^2 \theta \cos^2 \theta} =$

若 $\cos \theta \neq 0$ ，則 $\frac{\cos^2 \theta + \tan^2 \theta - 1}{\sin^2 \theta + \cos^2 \theta - \tan^2 \theta \cos^2 \theta} =$

- A. $\sin^4 \theta$ B. $\cos^4 \theta$ C. $-\sin^4 \theta$ D. $-\cos^4 \theta$
 E. $\tan^4 \theta$

3. If $x \neq 0$, $\log A$ denotes $\log_{10} A$, and $\log 5x^2 = a$, then $\log \frac{4}{x^4} =$

若 $x \neq 0$ ， $\log A$ 表示 $\log_{10} A$ ，及 $\log 5x^2 = a$ ，則 $\log \frac{4}{x^4} =$

- A. $2 - a$ B. $2 - 2a$ C. $2 - a^2$ D. $4 - a^2$
 E. $4 - 2a$

4. The largest value of $\frac{1}{5 - 2^{1 - \cos x}}$ is

$\frac{1}{5 - 2^{1 - \cos x}}$ 的最大值是

- A. 0 B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$ E. 1

5. In a school, there are only two Form 5 classes, namely, 5A and 5B. There are 35 and 40 students in 5A and 5B respectively. If 60% of the students in 5A and 75% of the students in 5B passed the same examination, what is the passing percentage of the whole Form 5 in that examination?

某學校的中五級只有兩班：5A 和 5B。5A 班和 5B 班分別有學生 35 和 40 人。若 60% 的 5A 班學生和 75% 的 5B 班學生在同一個考試中及格，整個五年級的學生於該考試的及格百份率是多少？

- A. 62.5% B. 67.5% C. 68% D. 70%
E. 72.5%

6. If the three points $(1, 3)$, $(3, 7)$ and $(4, k)$ are collinear, then $k =$
若 $(1, 3)$ 、 $(3, 7)$ 和 $(4, k)$ 三點為共線，則 $k =$

- A. 8 B. 9 C. 10 D. 11 E. 12

7. The weights of twelve girls form an arithmetic sequence. If the average weight of the three heaviest girls is 62.5kg and that of the five lightest girls is 42.5kg, the average weight of the remaining four girls is

十二名女孩的體重組成等差數列。若最重的三名女孩的平均體重是 62.5kg，而最輕的五名女孩的平均體重是 42.5 kg，那麼餘下四名女孩的平均體重是

- A. 52.5 kg B. 54.5 kg C. 51.75 kg D. 53.75 kg
E. 55.0 kg

8. Given that a, b, c, d, e, f is an arithmetic sequence. If $a + d = 13$, $c + e = 22$, find the sum of the terms in this sequence.

已知 a, b, c, d, e, f 是一個等差數列。若 $a + d = 13$ ， $c + e = 22$ ，求這個數列內各項之總和。

- A. 50 B. 51 C. 54 D. 57 E. 60

9. Which of the following equations has/have two distinct real roots?

以下哪個方程有兩個不同的實根?

I. $x(x-3) = 5(x-3)$

II. $(x-1)^2 = 9$

III. $4(2x+3)^2 = 0$

- | | |
|--------------------|-------------|
| A. III only | 只有 III |
| B. I and II only | 只有 I 和 II |
| C. I and III only | 只有 I 和 III |
| D. II and III only | 只有 II 和 III |
| E. I, II and III | I、II 和 III |

10. A number is first increased by 10% and then decreased by 10%. As a result of these two changes, the number is

某數首先增大 10%，然後再減少 10%。結果，這個數

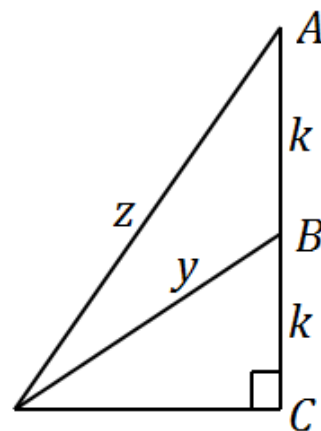
- | | |
|---------------------|---------|
| A. unchanged | 維持不變 |
| B. increased by 1% | 增大了 1% |
| C. increased by 11% | 增大了 11% |
| D. decreased by 1% | 減少了 1% |
| E. decreased by 11% | 減少了 11% |

11. In the figure, the three points A , B and C are collinear.

Then $z^2 - y^2 =$

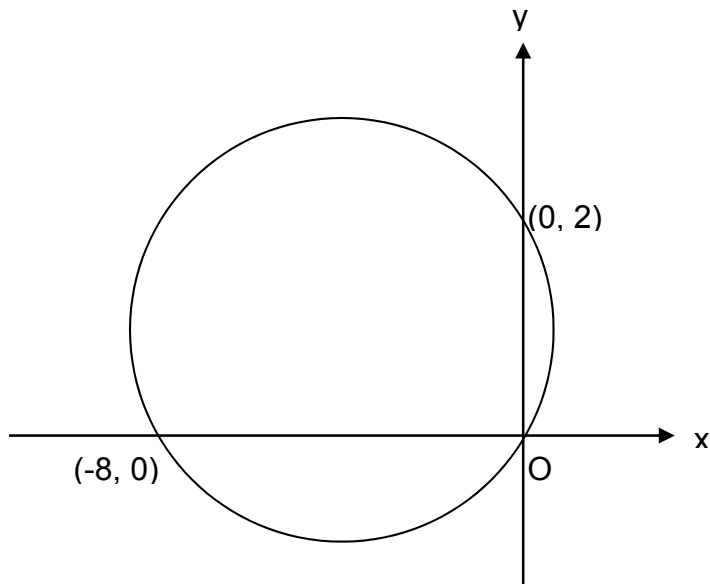
在圖中 A 、 B 和 C 三點為共線。則 $z^2 - y^2 =$

- | |
|-----------|
| A. k^2 |
| B. $2k^2$ |
| C. $3k^2$ |
| D. $4k^2$ |
| E. $5k^2$ |



12. In the figure, the circle passes through the two points $(-8, 0)$ and $(0, 2)$, and the origin O . Find the equation of the circle.

在圖中，圓穿過 $(-8, 0)$ 和 $(0, 2)$ 兩點及原點 O 。求該圓的方程。



- A. $x^2 + y^2 + 2x - 8y = 0$ B. $x^2 + y^2 - 4x + y = 0$
- C. $x^2 + y^2 + 4x - y = 0$ D. $x^2 + y^2 + 6x + 6y - 16 = 0$
- E. $x^2 + y^2 + 8x - 2y = 0$
13. 50 apples are shared between A and B . Given that the number of apples obtained by A is at least twice that obtained by B . Find the maximum number of apples obtained by B .
- 把 50 個蘋果分給 A 和 B 二人，已知 A 所得的蘋果數目最少是 B 的兩倍。求 B 最多可得到多少個蘋果？
- A. 16
B. 17
C. 18
D. 20
E. 33

14. If $(\sqrt{3} - 1)\sqrt{x} = 2$, then $x =$

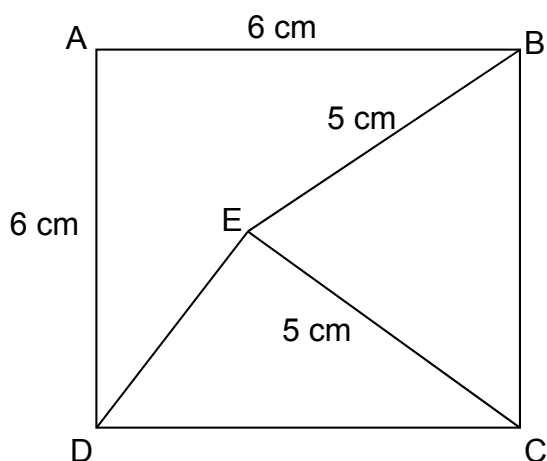
若 $(\sqrt{3} - 1)\sqrt{x} = 2$ ，則 $x =$

A. $4 + 2\sqrt{3}$ B. $4 - 2\sqrt{3}$ C. $\pm(4 + 2\sqrt{3})$

D. $\pm(4 - 2\sqrt{3})$ E. $-4 + 2\sqrt{3}$

15. In the figure, $ABCD$ is a square with side 6 cm. If $BE = CE = 5$ cm, find the length of ED .

在圖中， $ABCD$ 為一邊長 6 cm 的正方形。若 $BE = CE = 5$ cm，求 ED 的長度。



A. 6 cm B. 10 cm C. $\sqrt{13}$ cm

D. $2\sqrt{13}$ cm E. $13\sqrt{2}$ cm

16. Given that the median of a set of data 2, 2, 6, 6, 6, 8 and x is equal to their mean.

Find x .

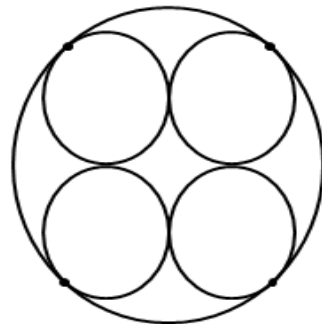
已知一組數據 2、2、6、6、6、8 和 x 的中位數等於它的平均數，求 x 。

A. 12 B. 10 C. 8 D. 6

E. Not able to find the result 無法計算結果

17. In the figure, the larger circle is tangent to the other four smaller circles. If the diameter of each of the four smaller circles is 2 cm, what is the area of the larger circle?

圖中大圓與其餘 4 個小圓相切。若 4 個小圓的直徑都為 2 cm，大圓的面積為多少？



- A. $(2 + 2\sqrt{3}) \cdot \pi \text{ cm}^2$ B. $(3 + 2\sqrt{2}) \cdot \pi \text{ cm}^2$
- C. $9\pi \text{ cm}^2$ D. $(2 + 3\sqrt{2}) \cdot \pi \text{ cm}^2$
- E. $(3 + \sqrt{2}) \cdot \pi \text{ cm}^2$
18. In a regular n -sided polygon, each interior angle is 3 times of each exterior angle. Find n .
某正 n 邊形的內角是每個外角的 3 倍，求 n 。
- A. 3 B. 4 C. 5 D. 6 E. 8
19. If $a + b = 3$ and $a^3 + b^3 = 18$, then $ab =$
若 $a + b = 3$ 及 $a^3 + b^3 = 18$ ，則 $ab =$
- A. 1 B. 3 C. 6 D. 8 E. None of these
以上皆非
20. Mr. Chan originally had \$10,000. He then borrowed \$ x from a bank at a simple interest rate of 5% per annum. He invested both sums of money for one year and the percentage of return was 3%. After he had returned the amount to the bank, he had \$9,800 left. Find x .
陳先生原有現金\$10,000，他再向銀行借了 \$ x ，按單利息計算，年利率為 5%。他將兩筆錢一起用作投資，一年後，陳先生在投資上的回報為 3%。他向銀行還清款項後，發覺尚餘現金\$9,800。求 x 。
- A. 10,000 B. 15,000 C. 20,000 D. 25,000
E. 35,000

2015/2016 學年 參考答案 MODEL ANSWER

Part I

1. (a) If $a : b : c = 2 : 5 : 3$, let $a = 2k, b = 5k, c = 3k$, where k is a nonzero constant.

$$\text{Then } \frac{1}{a} = \frac{1}{2k}, \frac{1}{b} = \frac{1}{5k}, \frac{1}{c} = \frac{1}{3k}.$$

$$\frac{1}{a} : \frac{1}{b} : \frac{1}{c} = \frac{1}{2k} : \frac{1}{5k} : \frac{1}{3k} = \frac{1}{2} : \frac{1}{5} : \frac{1}{3} = 15 : 6 : 10.$$

- (b) Notice that $-\cos 20^\circ = -\sin 70^\circ = \sin(180^\circ + 70^\circ) = \sin(360^\circ - 70^\circ)$, so

$$\sin \theta = -\cos 20^\circ \Rightarrow \sin \theta = \sin(180^\circ + 70^\circ), \text{ or } \sin \theta = \sin(360^\circ - 70^\circ)$$

$$\Rightarrow \theta = 180^\circ + 70^\circ, \text{ or } 360^\circ - 70^\circ.$$

$$\therefore \theta = 250^\circ, \text{ or } 290^\circ.$$

2. (a) By solving the equations $4x + 3y - 96 = 0$ and $3x - 4y + 28 = 0$

simultaneously, it was found that the coordinates of C are $(12, 16)$. The x -coordinate of A could be found by plugging $y = 0$ into $4x + 3y - 96 = 0$, obtaining $x = 24$. Similarly, the y -coordinate of B was found to be 7.

$$\text{Slope of } AB = \frac{0 - 7}{24 - 0} = -\frac{7}{24}.$$

So the slope of the required line perpendicular to AB is $\frac{24}{7}$.

The equation of the line passing through C and perpendicular to AB is

$$y - 16 = \frac{24}{7}(x - 12), \text{ which simplifies to } 24x - 7y - 176 = 0.$$

(b) $(\text{Slope of } BC) \times (\text{Slope of } AC) = \frac{3}{4} \times \left(-\frac{4}{3}\right) = -1 \Rightarrow BC \perp AC$. Thus $\triangle BCA$

is a right angled triangle, and therefore

$$\begin{aligned} BC &= \sqrt{AB^2 - AC^2} \\ &= \sqrt{OA^2 + OB^2 - AC^2} \\ &= \sqrt{7^2 + 24^2 - (24 - 12)^2 - (0 - 16)^2} \\ &= 15. \end{aligned}$$

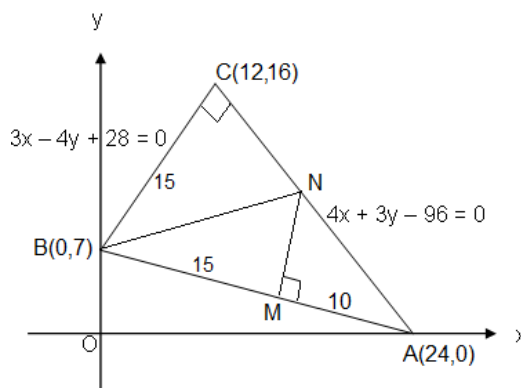
Also,

$$BM = AB - MA = \sqrt{7^2 + 24^2} - 10 = 15.$$

$$\therefore CN = \sqrt{BN^2 - BC^2} = \sqrt{BN^2 - 15^2}$$

$$MN = \sqrt{BN^2 - BM^2} = \sqrt{BN^2 - 15^2}$$

$$\Rightarrow CN = MN.$$



3. (a) When $S \leq 6$, let $P = k$, and when $S > 6$, let $P = k + m(S - 6)$, where both k and m are constants.

Given that $P = 128$ if $S = 8$, and $P = 185$ if $S = 11$, we have

$$\begin{cases} k + m(8 - 6) = 128 \\ k + m(11 - 6) = 185 \end{cases} \Rightarrow m = 19, \text{ and } k = 90.$$

$$\text{Therefore, } P = \begin{cases} 90 & \text{if } S \leq 6; \\ 90 + 19(S - 6) & \text{if } S > 6. \end{cases}$$

- (b) Obviously $S > 6$ for customer A . Because A paid more than the total amount B and C paid, this requires

$$P = 90 + 19(S - 6) > 128 + 185$$

$$\Rightarrow S > \frac{337}{19} = 17 \frac{14}{19}$$

So A had eaten at least 18 pieces of sushi.

4. (a) By applying long division, it was found that

$$x^3 - 2x^2 - 7x + 2 = (x - 2)(x^2 - 7) - 12,$$

$$\therefore Q(x) = x^2 - 7, \quad R = -12.$$

- (b) If $x^2 - 7 \equiv A(x-2)^2 + B(x-2) + C$, then
 $x^2 - 7 \equiv A(x^2 - 4x + 4) + B(x-2) + C \equiv Ax^2 + (B-4A)x + (4A-2B+C)$.
 By comparing coefficients, we have

$$\begin{cases} A = 1 \\ B - 4A = 0 \\ 4A - 2B + C = -7 \end{cases} \Rightarrow A = 1, B = 4, C = -3.$$

- (c) $f(x) = (x-2)(x^2 - 7) - 12$ Result in Part (a)
 $= (x-2)[(x-2)^2 + 4(x-2) - 3] - 12$ Result in Part (b)
 $= (x-3)^3 + 4(x-2)^2 - 3(x-2) - 12$

5. (a) All single outcomes are equally likely, and are given in the table below:

		Bag B					
		Banknote	\$20	\$50	\$100	\$500	\$1,000
Bag A	\$10	\$30	\$60	\$110	\$510	\$1,010	
	\$20	\$40	\$70	\$120	\$520	\$1,020	
	\$50	\$70	\$100	\$150	\$550	\$1,050	
	\$100	\$120	\$150	\$200	\$600	\$1,100	

Number of possible outcomes = 20

Number of favorable outcomes (shaded cells) = 10

Required probability = $\frac{10}{20} = \frac{1}{2}$.

- (b) The event of getting a total face values more than \$100 but less than \$500 consists of the single outcomes given in the table below:

		Bag B					
		Banknote	\$20	\$50	\$100	\$500	\$1,000
Bag A	\$10	\$110					
	\$20	\$120					
	\$50	\$150					
	\$100	\$120	\$150	\$200			

Number of possible outcomes = 6

Number of favorable outcomes = 1 (the shaded cell)

$$\text{Required probability} = \frac{1}{6}$$

Part II

1	2	3	4	5	6	7	8	9	10
E	E	B	E	C	B	D	D	B	D

11	12	13	14	15	16	17	18	19	20
C	E	A	A	C	A	B	E	A	D

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Part I

1. Solve the following equations for real solution x :

求下列方程的實數解 x :

(a) (4 marks 4 分) $\frac{x+1}{x-1} = \frac{x+2}{3}$

(b) (4 marks 4 分) $\frac{x-7}{2} = \sqrt{x-4}$

2. (a) (4 marks) Solve the equation $2^{x+2} + 5 \cdot 2^x = 72$.

(4 分) 解方程 $2^{x+2} + 5 \cdot 2^x = 72$ 。

- (b) (4 marks) Two times the age of Sam 3 years later is 2 years more than four times his age 3 years ago. What is Sam's present age?

(4 分) 森 3 年後的年齡的兩倍，比 3 年前的年齡的四倍多 2 歲。求森現時的年齡。

3. A mobile phone company offers two usage plans, A and B . Plan A charges \$0.8 per minute of usage. In Plan B , the monthly charge is the sum of two parts: the first part is a fixed service charge, while the second part varies directly with the monthly usage (x minutes). If the monthly usage is 100 minutes, the monthly charge under Plan B is \$100; if the monthly usage is 200 minutes, the monthly charge is \$150.

某流動電話公司提供 A 、 B 兩個通話計劃。 A 計劃每分鐘收費 \$0.8。在 B 計劃中，每月的收費是兩部份之和：首部份是一筆固定的服務費，而次部份則隨每月用量 (x 分鐘) 正變。若每月用量為 100 分鐘，則在 B 計劃下該月收費是 \$100；若每月用量為 200 分鐘，則月費是 \$150。

- (a) (4 marks) Find the monthly charge in terms of x .

(4 分) 以 x 表示每月的收費。

- (b) (4 marks) For what monthly usage will it be more desirable to use Plan B than Plan A ?

(4 分) 每月用量為多少會使得 B 計劃比 A 計劃划算？

4. There are 12 boys and 8 girls in a class of students. Three of them are chosen at random from the class to form a committee.

某班有 12 名男生和 8 名女生。現隨機選出三名學生組成委員會。

- (a) (3 marks) Find the probability that 3 boys will be selected.

(3 分) 求三名委員都是男生的概率(機率)。

- (b) (3 marks) Find the probability that 2 boys and 1 girl will be selected.

(3 分) 求三名委員中有兩名男生和一名女生的概率。

- (c) (3 marks) Find the probability that at least 1 girl will be selected.

(3 分) 求三名委員中有最少一名女生的概率。

5. (a) (3 marks) How many integers greater than ($>$) 400 but less than ($<$) 600 are multiples of either 3 or 7? Note: these include numbers like 441 and 525, which are multiples of both 3 and 7.

(3 分) 大於($>$) 400, 但小於($<$) 600 的整數中, 有多少個是 3 或 7 的倍數? 注意: 同時是 3 和 7 的倍數, 如 441 和 525 等數字亦包括在內。

- (b) (4 marks) Find the sum of those integers that are greater than($>$) 400 but less than($<$) 600, and which are multiples of 7.

(4 分) 求大於($>$) 400, 但小於($<$) 600, 而又是 7 的倍數的整數之和。

Part 2

1. 1. Given $S = \frac{n}{2}[2a + (n-1)d]$, then $a =$

已知 $S = \frac{n}{2}[2a + (n-1)d]$, 則 $a =$

A. $\frac{S}{n} - \frac{d}{2} + \frac{nd}{2}$

B. $\frac{S}{n} + \frac{d}{2} - \frac{nd}{2}$

C.

$\frac{S}{n} - \frac{d}{2} - n + 1$

D. $\frac{2S}{n} - \frac{d}{2} - nd$

E. $\frac{2S}{n} + \frac{d}{2} + nd$

2. A supermarket raises the selling price of a brand of soft drinks by 20%. However, the sales have dropped so much that the supermarket later decides to offer a discount so that the selling price is the same as that before the price increase. Find the discount percentage.

一間超級市場把某牌子汽水的售價提高 20% 後，銷售量大跌，因此決定要提供折扣，使得折扣後的售價和加價前的售價相同。求折扣百分率。

- A. 80% B. 25% C. 20% D. $17\frac{1}{3}\%$ E. $16\frac{2}{3}\%$

3. If $a : b = 2 : 3$, then $(2a + b) : (a + 2b) =$
若 $a : b = 2 : 3$ ，則 $(2a + b) : (a + 2b) =$

- A. 2 : 3 B. 3 : 2 C. 5 : 6 D. 7 : 8 E. 8 : 7

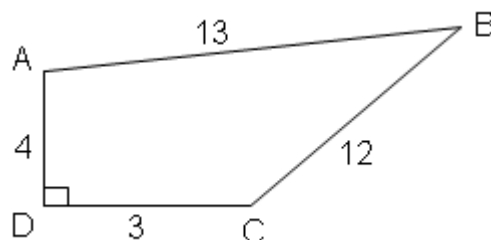
4. If the graph of $y = f(x)$ is reflected in the x -axis, and is then translated 1 unit vertically upward to become the graph of $y = g(x)$, then $g(x) =$

若 $y = f(x)$ 的圖像沿 x 軸反射後再沿 y 軸方向向上平移 1 單位，變換成 $y = g(x)$ 的圖像，則 $g(x) =$

- A. $f(x) + 1$ B. $f(x) - 1$ C. $1 - f(x)$ D. $f(x + 1)$ E. $f(x - 1)$

5. In the figure, the area of $ABCD$ is
在圖中， $ABCD$ 的面積是

- A. 32
B. 36
C. 54
D. 72
E. 84



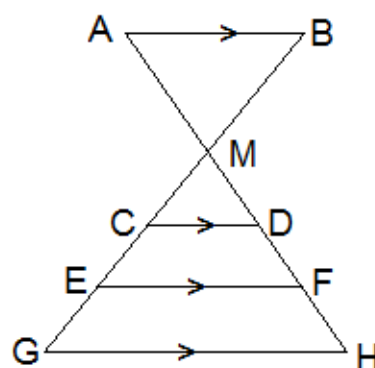
6. Find the value of the constant c if $\sum_{n=1}^{\infty} (3-c)^{-n} = \frac{1}{3-c} + \frac{1}{(3-c)^2} + \dots = 4$.

若 $\sum_{n=1}^{\infty} (3-c)^{-n} = \frac{1}{3-c} + \frac{1}{(3-c)^2} + \dots = 4$ ，求常數 c 的值。

- A. $\frac{3}{2}$ B. $\frac{8}{3}$ C. $\frac{5}{3}$ D. $\frac{7}{4}$ E. none of these 以上皆不對

7. In the figure, the four line segments AB, CD, EF, and GH are parallel, and AH and BG intersect at M. How many pair(s) of similar triangles can be found in the figure?

圖中線段 AB、CD、EF 及 GH 互相平行，而 AH 與 BG 相交於 M。問共有多少對相似三角形？



- A. 2 pair 2 對
 B. 3 pairs 3 對
 C. 4 pairs 4 對
 D. 5 pairs 5 對
 E. 6 pairs 6 對
8. If the mid-point of $(1, a)$ and $(b, 2)$ is $(3, 4)$, then the mid-point of $(b, 5)$ and $(6, a)$ is
 若 $(1, a)$ 和 $(b, 2)$ 的中點是 $(3, 4)$ ，則 $(b, 5)$ 和 $(6, a)$ 的中點是
- A. $(5, 6)$ B. $(5.5, 5.5)$ C. $(6, 5)$
 D. $(6.5, 7.5)$ E. $(8, 7)$

9. If $4x^3 + 2x^2 + ax + 6$ is divided by $2x + 3$, the remainder is $\frac{9}{2}$. Find the constant a .

若 $4x^3 + 2x^2 + ax + 6$ 除以 $2x + 3$ 的餘式(餘數)是 $\frac{9}{2}$ ，求常數 a 。

- A. 5 B. -5 C. 0 D. 3 E. -3

10. In a class, the number of boys and girls are in ratio 3 : 2. In a test, the mean score for boys is 70 while the mean score for the whole class is 76. The mean score for girls is 某班中男生和女生人數的比例是 3 : 2。在一次測驗中，男生的平均分數是 70，而全班的平均分數是 76。女生的平均分數是

A. 72.4 B. 73.6 C. 80 D. 82 E. 85

11. Solve the inequalities $1 + 2x < 1 - x < -1 + 3x$.

解不等式 $1 + 2x < 1 - x < -1 + 3x$ 。

A. $0 < x < \frac{1}{2}$ B. $x < 0$ or $x > \frac{1}{2}$ $x < 0$ 或 $x > \frac{1}{2}$
 C. $\frac{1}{2} < x < 1$ D. $x > \frac{1}{2}$ E. No solution 沒有解

12. Simplify $\sin(270^\circ - A)\cos(270^\circ + A)$.

化簡 $\sin(270^\circ - A)\cos(270^\circ + A)$ 。

A. $\sin A \cos A$ B. $-\sin A \cos A$ C. $\sin^2 A$
 D. $-\cos^2 A$ E. none of these 以上皆不對

13. The minimum value of $1 - 2\cos 3x$ is

$1 - 2\cos 3x$ 的極小值為

A. -5 B. -2 C. -1 D. 3 E. $-\sqrt{5}$

14. Let $A = (-2, 4)$ and $B = (3, -1)$. The straight line AB meets the x -axis at P . Find the ratio $AP : PB$.

設 $A = (-2, 4)$ 和 $B = (3, -1)$ 。直線 AB 和 x 軸相交於 P 。求比值 $AP : PB$ 。

A. 1 : 4 B. 1 : 5 C. 2 : 3 D. 3 : 2 E. 4 : 1

15. If (m, n) is a point on the circle $x^2 + y^2 - 2x - 4y - 4 = 0$, the smallest possible value of m is

若 (m, n) 是圓 $x^2 + y^2 - 2x - 4y - 4 = 0$ 上的點，則 m 可取的最小值是

- A. -3 B. -2 C. -1 D. 0 E. 1

16. Find the standard deviation of the numbers $x-4$, $x-1$, x and $x+5$. Correct your answer to 1 decimal place if necessary.

求 $x-4$ 、 $x-1$ 、 x 和 $x+5$ 的標準差。如有需要，答案準確至小數點後 1 位。

- A. 2.5 B. 3.2 C. 4.7 D. 5.3 E. 10.5

17. Given that $f(x) = \frac{ax-1}{x^2+1}$. If $f(x) = f(-x)$ for all x , find $f(1)$.

已知 $f(x) = \frac{ax-1}{x^2+1}$ 。若 $f(x) = f(-x)$ 對所有 x 都成立，求 $f(1)$ 。

- A. 0 B. $-\frac{1}{2}$ C. -1 D. -2 E. 2

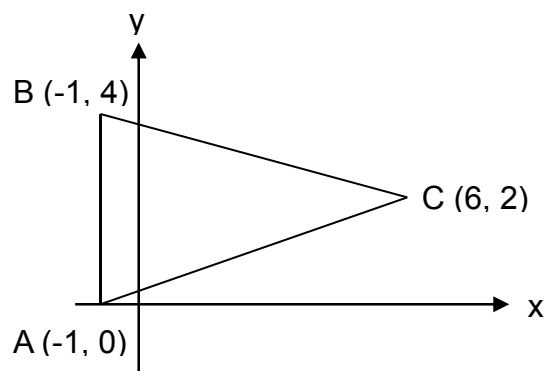
18. If $\log 2 = a$ and $\log 3 = b$, then $\log 45 =$

若 $\log 2 = a$ 和 $\log 3 = b$ ，則 $\log 45 =$

- A. $a-b+1$ B. $2b-a+1$ C. $a+2b-1$
D. $a+b+1$ E. $a-2b-1$

19. In the figure, the area of $\triangle ABC$ is
 在圖中， $\triangle ABC$ 的面積是

- A. 4
 B. 12
 C. 14
 D. 20
 E. 28

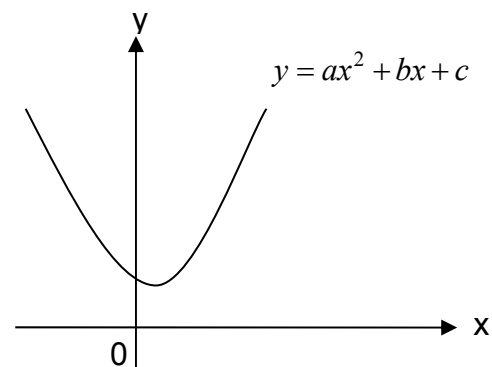


20. The figure shows the graph of $y = ax^2 + bx + c$.

Which of the following cannot be the values of a , b and c ?

圖中所示為 $y = ax^2 + bx + c$ 的圖像，下列何者不可能是 a 、 b 和 c 的值？

- | | a | b | c |
|----|-----|-----|-----|
| A. | 1 | -1 | 1 |
| B. | 1 | -2 | 3 |
| C. | 4 | -1 | 4 |
| D. | 1 | -4 | 4 |
| E. | 3 | -5 | 3 |



2014/2015 學年 參考答案 MODEL ANSWER

Part I

$$1. \quad (a) \quad \frac{x+1}{x-1} = \frac{x+2}{3}$$

$$3(x+1) = (x-1)(x+2)$$

$$3x+3 = x^2 + x - 2$$

$$x^2 - 2x - 5 = 0$$

$$x = \frac{2 \pm \sqrt{4+20}}{2} = 1 \pm \sqrt{6}, \text{ or } 3.449, -1.449.$$

$$(b) \quad \frac{x-7}{2} = \sqrt{x-4} \quad \dots\dots\dots (*)$$

Both sides of (*) are non-negative (≥ 0), implying $x \geq 7$.

On squaring both sides of (*), we have

$$\left(\frac{x-7}{2}\right)^2 = x-4$$

$$(x-7)^2 = 4 \cdot (x-4)$$

$$x^2 - 14x + 49 = 4x - 16$$

$$x^2 - 18x + 65 = 0$$

$$(x-13)(x-5) = 0$$

Therefore, the only real root of (*) is 13 ($x=5$ is rejected because $x \geq 7$).

$$2. \quad (a) \quad 2^{x+2} + 5 \cdot 2^x = 72$$

$$2^x \cdot 2^2 + 5 \cdot 2^x = 72$$

$$4 \cdot 2^x + 5 \cdot 2^x = 72$$

$$9 \cdot 2^x = 72$$

$$2^x = 8$$

$$x = 3$$

(b) Let Sam be now x years old.

$$2 \cdot (x+3) = 4 \cdot (x-3) + 2$$

$$2x + 6 = 4x - 10$$

$$2x = 16$$

$$x = 8$$

So Sam's present age is 8 years.

3. (a) Let the monthly charge be \$ C . Then $C = a + bx$, where a and b are constants.

We have

$$\begin{cases} 100 = a + b(100) \\ 150 = a + b(200) \end{cases}$$

By solving the above system of linear equations, we found that $a = 50$, $b = 0.5$.

So $C = 50 + 0.5x$.

- (b) If it is more desirable to use Plan B then Plan A , then

$$50 + 0.5x < 0.8x$$

$$0.3x > 50$$

$$x > 166\frac{2}{3}$$

Therefore, it is more desirable to use Plan B then Plan A if the monthly usage exceeds $166\frac{2}{3}$ (or 167) minutes.

4. (a) $P(3 \text{ boys will be selected}) = \frac{12}{20} \cdot \frac{11}{19} \cdot \frac{10}{18} = \frac{11}{57}$, or 0.193.

Alternatively, the above probability is $\frac{C_{12}^3 \cdot C_8^0}{C_{20}^3} = \frac{11}{57}$, where C_n^r denotes the number of ways of choosing r objects out of n objects without replacement.

- (b) $P(2 \text{ boys and 1 girl will be selected}) = 3 \cdot \left(\frac{12}{20} \cdot \frac{11}{19} \cdot \frac{8}{18} \right) = \frac{44}{95}$, or 0.463.

Alternatively, the required probability is $\frac{C_{12}^2 \cdot C_8^1}{C_{20}^3} = \frac{44}{95}$.

- (c)

$$P(\text{at least 1 girl will be selected}) = 1 - P(3 \text{ boys will be selected}) = 1 - \frac{11}{57} = \frac{46}{57} \text{ or}$$

0.807.

5. (a) Number of integers greater than 400 and less than 600 that are multiple of 3 is equal to the integral part of $\frac{599 - 401}{3}$, which is 66.

Number of integers greater than 400 and less than 600 that are multiple of 7 is the integral part of $\frac{599 - 401}{7}$, which is 28.

Number of integers greater than 400 and less than 600 that are multiple of both

3 and 7 is equal to the integral part of $\frac{599 - 401}{21}$, which is 9.

Thus the number of integers greater than 400 and less than 600 that are multiple of either 3 or 7, or both, is $66 + 28 - 9$, which is 85.

5. (b) Numbers lying within the specified range, and which are multiples of 7 are 406, 413, 420,, 595.

This is an arithmetic sequence with 28 terms. Its sum is

$$406 + 413 + 420 + \dots + 595 = \left(\frac{406 + 595}{2} \right) \cdot 28 = 14014.$$

Part II

- | | | | |
|-------|-------|-------|-------|
| 1. B | 2. E | 3. D | 4. C |
| 5. B | 6. D | 7. E | 8. B |
| 9. B | 10. E | 11. E | 12. B |
| 13. C | 14. E | 15. B | 16. B |
| 17. B | 18. B | 19. C | 20. D |