

座位編號： _____

澳門科技大學
2014/2015 學年度入學/分班考試

《數學》

姓名： _____ 身份證編號： _____

申請編號： 1409AP -B

報讀課程(第一志願)：(請 出適當者)

- | | | |
|---|--------------------------------------|-------------------------------------|
| <input type="checkbox"/> 理學學士學位課程 | <input type="checkbox"/> 工商管理學士學位課程 | <input type="checkbox"/> 法學學士學位課程 |
| <input type="checkbox"/> 中醫學學士學位課程 | <input type="checkbox"/> 生物醫學學士學位課程 | <input type="checkbox"/> 中藥學學士學位課程 |
| <input type="checkbox"/> 國際旅遊管理學士學位課程 | <input type="checkbox"/> 酒店管理學士學位課程 | <input type="checkbox"/> 餐飲管理學士學位課程 |
| <input type="checkbox"/> 藝術學-藝術設計學士學位課程 | <input type="checkbox"/> 新聞傳播學學士學位課程 | <input type="checkbox"/> 外國語學士學位課程 |

考試日期： _____ 考試時間： _____

注意事項：

1. 包括此頁在內中英文試題，本試卷共 6 版 3 頁。
2. 本試卷由兩部分組成：
 - ◆ 第一部份包括十道選擇題，每道題 5 分，共 50 分。
 - ◆ 第二部份包括五道計算題，每道題 10 分，共 50 分。這五道題要求寫上計算或證明步驟，否則將酌情扣分。
3. 所有答案寫在答題紙上，寫在其他地方的答案將不會獲評分。
4. 考生不准使用計算器。
5. 考生不得使用鉛筆或紅色筆作答，否則將不予評分。

以下部份由校方填寫：

第一部份得分：

題號	1	2	3	4	5	6	7	8	9	10	合計
得分											

第二部份得分：

題號	11	12	13	14	15	合計
得分						

全卷總分 核查結果

第一部分：選擇題

從四個選項中選擇一個正確答案，每題 5 分，共 50 分

1. 設集合 $A = \{a^2, a+1, -1\}$, $B = \{2a-1, |a-2|, 3a^2+4\}$, 若 $A \cap B = \{-1\}$, 則 a 的值是 ()
A. -1 B. 0 或 1 C. 2 D. 0
2. 設 $a > 1 > b > -1$, 則下列不等式中恆成立的是 ()
A. $\frac{1}{a} < \frac{1}{b}$ B. $\frac{1}{a} > \frac{1}{b}$ C. $a > b^2$ D. $a^2 > 2b$
3. 若方程 $x^2 + (m+2)x + m + 5 = 0$ 只有正根, 則 m 的取值範圍是 ()
A. $m \leq -4$ 或 $m \geq 4$ B. $-5 < m \leq -4$ C. $-5 \leq m \leq -4$ D. $-5 < m < -2$
4. 函數 $f(x) = \frac{3x^2}{\sqrt{1-x}} + \ln(3x+1)$ 的定義域是 ()
A. $(-\infty, -\frac{1}{3})$ B. $(-\frac{1}{3}, \frac{1}{3})$ C. $(-\frac{1}{3}, 1)$ D. $(-\frac{1}{3}, +\infty)$
5. 等差數列 $\{a_n\}$ 中, $a_1 + a_5 = 10, a_4 = 7$, 則數列 $\{a_n\}$ 的公差為 ()
A. 1 B. 2 C. 3 D. 4
6. 已知 $\frac{x^2}{m-3} + \frac{y^2}{1-m} = 1$ 為一雙曲線方程, 則 m 的取值範圍是 ()
A. $m > 3$ B. $m < 1$ C. $1 < m < 3$ D. $m < 1$ 或 $m > 3$
7. 已知二次函數 $y = a(x-1)^2 + b$ 有最小值 -1 , 則 a 與 b 之間的大小關係是 ()
A. $a < b$ B. $a = b$ C. $a > b$ D. 不能確定
8. a, b, c, d, e 共 5 個人, 從中選 1 名組長 1 名副組長, 但 a 不能當副組長, 不同的選法總數是 ()
A. 20 B. 1 C. 16 D. 6
9. 設命題 M 為: $0 < x < 5$, 命題 N 為 $|x-2| < 3$, 那麼 M 是 N 的 ()
A. 充分不必要條件 B. 必要不充分條件
C. 既不充分也不必要條件 D. 充要條件
10. 已知 $(\frac{1}{3})^{\tan(\theta-\pi)} > 1$, 則 θ 是 ()
A. 第一或第二象限 B. 第二或第四象限
C. 第一或第三象限 D. 第二或第三象限

第二部分：計算題

要求寫出必要計算或證明步驟，否則將酌情扣分，每題 10 分，共 50 分

11. 已知 $x \in [-3, 2]$ ，求 $y = \frac{1}{4^x} - \frac{1}{2^x} + 1$ 的最大值與最小值。
12. 已知 $f(x) = 5 \sin x \cos x - 5\sqrt{3} \cos^2 x + \frac{5}{2}\sqrt{3}, (x \in R)$
- (1) 求 $f(x)$ 的最小正週期及最值；
 - (2) 求 $f(x)$ 的單調區間。
13. 數列 $\{a_n\}$ 的前 n 項和記為 $S_n, a_1 = 1, a_{n+1} = 2S_n + 1 (n \geq 1)$ ，求 $\{a_n\}$ 的通項公式。
14. 已知點 $P(-3, -4)$ 和直線 $l_1: x + 2y - 4 = 0$ ，
- (1) 求過 P 點且與直線 l_1 垂直的直線 l_2 的方程；
 - (2) 設直線 l_1 與直線 l_2 的交點為 Q ，求以線段 PQ 為直徑的圓 C 的方程。
15. 試比較 2^n 與 $(n+1)^2$ 的大小 ($n \in N^*, n \geq 6$)，並用數學歸納法證明你的結論。

Seat No.: _____

Macau University of Science & Technology Admission/Placement Examination 2014/2015

《Mathematics》

Name of Student: _____ ID No.: _____

Application No.: 1409AP -B

Program applied for 1st choice (Please the appropriate)

- | | |
|---|---|
| <input type="checkbox"/> Bachelor of Science | <input type="checkbox"/> Bachelor of Business Administration |
| <input type="checkbox"/> Bachelor of Laws | <input type="checkbox"/> Bachelor of Traditional Chinese Medicine |
| <input type="checkbox"/> Bachelor of Biomedicine | <input type="checkbox"/> Bachelor of Pharmacy in Chinese Medicine |
| <input type="checkbox"/> Bachelor of International Tourism Management | <input type="checkbox"/> Bachelor of Hotel Management |
| <input type="checkbox"/> Bachelor of Food and Beverage Management | <input type="checkbox"/> Bachelor of Arts in Art Design |
| <input type="checkbox"/> Bachelor of Arts in Journalism and Communication | <input type="checkbox"/> Bachelor of Arts in Foreign Language Studies |

Date: _____

Time allowed: _____

Instructions:

1. There are 6 pages including this page, exam questions in Chinese and English.
2. There are two parts in this examination paper:
 - Part I consists of 10 multiple-choice questions worth 50 points in total, 5 points each.
 - Part II consists of 5 questions worth 50 points in total, 10 points each. Show all your steps or proofs in getting the answers. Full credits will be given only if the answer and all steps are correct and clearly shown.
3. **Put all your answers in answer sheets provided. Answers put elsewhere will not be marked.**
4. **Students are not allowed to use calculator.**
5. **Don't use pencil or red-pen to give answers, otherwise they will not be marked.**

This part is used by the University for marking purpose:

Part I:

No.	1	2	3	4	5	6	7	8	9	10	Sum
Score											

Part II:

No.	11	12	13	14	15	Sum
Score						

Total

Checked

Part 1: Multiple-Choice

Choose the best answer to each question, 5 points each, 50 points total.

1. Suppose that the sets $A = \{a^2, a+1, -1\}$, $B = \{2a-1, |a-2|, 3a^2+4\}$ and $A \cap B = \{-1\}$, then a will be ()
A. -1 B. 0 or 1 C. 2 D. 0
2. Let $a > 1 > b > -1$. Which of the four inequalities is valid ()
A. $\frac{1}{a} < \frac{1}{b}$ B. $\frac{1}{a} > \frac{1}{b}$ C. $a > b^2$ D. $a^2 > 2b$
3. If the equation $x^2 + (m+2)x + m + 5 = 0$ has only positive solutions, then the range of the values of m is ()
A. $m \leq -4$ or $m \geq 4$ B. $-5 < m \leq -4$ C. $-5 \leq m \leq -4$ D. $-5 < m < -2$
4. The domain of the function $f(x) = \frac{3x^2}{\sqrt{1-x}} + \ln(3x+1)$ is ()
A. $(-\infty, -\frac{1}{3})$ B. $(-\frac{1}{3}, \frac{1}{3})$ C. $(-\frac{1}{3}, 1)$ D. $(-\frac{1}{3}, +\infty)$
5. Assume that in the arithmetic progression $\{a_n\}$, $a_1 + a_5 = 10$, $a_4 = 7$. Then the common difference of $\{a_n\}$ is ()
A. 1 B. 2 C. 3 D. 4
6. Suppose $\frac{x^2}{m-3} + \frac{y^2}{1-m} = 1$ is the equation of a hyperbola, then m satisfies ()
A. $m > 3$ B. $m < 1$ C. $1 < m < 3$ D. $m < 1$ or $m > 3$
7. Assume that the quadratic function $y = a(x-1)^2 + b$ has an absolute minimum value -1 . Then the relation of a and b is ()
A. $a < b$ B. $a = b$ C. $a > b$ D. Indeterminable
8. There are five people, labeled as a, b, c, d, e . Please choose one person as the group leader and another person as the associated group leader. Require that a cannot be the associated group leader. The total number of different choices is ()
A. 20 B. 10 C. 16 D. 6
9. Assume that the statement M is $0 < x < 5$, and the statement N is $|x-2| < 3$. Then M is () of N.
A. a sufficient condition B. a necessary condition
C. neither sufficient nor necessary condition D. a sufficient and necessary condition
10. Suppose that $(\frac{1}{3})^{\tan(\theta-\pi)} > 1$, then the terminal side of θ is in ()
A. the first quadrant or the second quadrant
B. the second quadrant or the fourth quadrant
C. the first quadrant or the third quadrant
D. the second quadrant or the third quadrant

Part II: Calculations

Show all your steps or proofs in getting the answers. Full credits will be given only if the answer and all steps are correct and clearly shown, 10 points each, 50 points total.

11. Find the absolute extrema of the function $y = \frac{1}{4^x} - \frac{1}{2^x} + 1$ for $x \in [-3, 2]$.

12. Let $f(x) = 5 \sin x \cos x - 5\sqrt{3} \cos^2 x + \frac{5}{2}\sqrt{3}$, ($x \in R$).

(1) Find the minimum positive period and absolute extrema of $f(x)$;

(2) Find the monotone intervals of $f(x)$.

13. Let S_n denote the sum of first n terms of progression $\{a_n\}$. Assume that

$a_1 = 1$, $a_{n+1} = 2S_n + 1$ ($n \geq 1$). Find the general term formula of $\{a_n\}$.

14. Given point $P(-3, -4)$ and line $l_1 : x + 2y - 4 = 0$,

(1) Find line l_2 which passes through P , and is perpendicular to l_1 ;

(2) Suppose Q is the intersection point of l_1 and l_2 . Find the equation of circle C with segment PQ as its diameter.

15. Compare 2^n and $(n+1)^2$ ($n \in N^*$, $n \geq 6$), and prove your conclusion by mathematical induction.